MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXXXIII. NOVEMBER 10, 1922. NO. 1

Professor A. S. EDDINGTON, F.R.S., PRESIDENT, in the Chair.

Ruby Élise Colomb, Lecturer in Mathematics, Birmingham University;
Ralph Howard Fowler, M.A., Fellow of Trinity College, Cambridge;
Captain H. Lloyd Jenkins, R.N.R., Senior Surveyor, Board of Trade, Newport, Mon.;
D. Vaughan Johnston, M.A., Principal, Lewis' School, Pengam, via Cardiff;
Earle Garfield Linsley, Professor of Geography and Astronomy in Mills College, Oakland, California;
Thomas A. Lumsden, M.A., B.Sc., Assistant Lecturer in Mathematics, Birmingham University;
Christopher Thomas Preece, Lecturer in Mathematics, Birmingham University;
Dr. Frank E. Ross, Research Laboratory, Eastman Kodak Company, Rochester, N.Y., U.S.A.;
Huish Webber, The Vicarage, Studham, near Dunstable, Beds; and
Ernest Wunderlich, St. Omer, Waverley, N.S.W., Australia,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

John Campbell Begg, 12 Fifield Street, Roslyn, Dunedin, New Zealand (proposed by C. E. Adams);
William Lionel Edgar Bennett, B.A., B.Sc., Christ's College, Cambridge, and 57 Lansdowne Street, Hove, Sussex (proposed by Horace Lamb);

Lieut.-Col. Frank Brown, R.A.O.C., M.I.M.E., 8 Bower Mount Road, Maidstone (proposed by T. Preston Battersby);

Prof. Julius Arthur Brown, Director of the Observatory of the American University, Beirut, Syria (proposed by H. H. Joy);

Claude DelaCherois Crommelin, B.A., 55 Ulundi Road, Blackheath, London, S.E. 3 (proposed by A. C. D. Crommelin);

Capt. William Ellery, 68 Victoria Street, London, S.W. 1 (proposed by C. Eddie);

Frank Fairchild Fergusson, A.M.I.C.E., F.R.G.S., P.O. Box 2217, Temple Chambers, Calcutta (proposed by W. Newbold);

Kripa Shankar Hajela, M.Sc., LL.B., LL.M., M.R.A.S., Reader in Law, Lucknow University, 22 Kutchery Road, Lucknow, India (proposed by K. B. Madhava);

David Laugharne-Thornton, Magdalene College, Cambridge, and Cross Banks, Shipley, Yorks (proposed by A. S. Eddington);

Lady Lockyer, Sidmouth (proposed by F. W. Dyson);

Thomas Logie MacDonald, Asst. Sec., West of Scotland Branch of the British Astronomical Association, 9 Colebroke Terrace, Glasgow, W. (proposed by J. J. Ross);

Neal M'Neill, F.R.G.S., F.R.C.I., 29 Newman Street, London, W. 1 (proposed by A. F. Butcher);

Capt. J. Marden, Stodham Park, Liss, Hants (proposed by A. C. D. Crommelin);

Ross W. Marriott, Ph.D., Asst. Professor of Mathematics, Sproul Observatory, Swarthmore College, Swarthmore, Penn., U.S.A. (proposed by L. J. Comrie);

William Milburn, Wolsingham Observatory, Tow Law, Co. Durham (proposed by T. H. E. C. Espin);

Eric Harold Neville, M.A., Professor of Mathematics, University College, Reading, 160 Castle Hill, Reading (proposed by W. J. Greenstreet);

Major J. A. Pearce, M.A., University Fellow, Lick Observatory, Mount Hamilton, California, U.S.A. (proposed by F. C. Leonard);

John H. Pitman, M.A., Asst. Professor of Mathematics and Astronomy, Sproul Observatory, Swarthmore College, Swarthmore, Penn., U.S.A. (proposed by J. A. Miller);

Capt. Richard John Porter, Board of Trade Surveyor, Belfast (proposed by E. J. Sharpe);

Jacobus Cornelius Reyneke, Rocklands, Wepener, O.F.S., S. Africa (proposed by R. T. A. Innes);

William Nicholas Roseveare, M.A., Professor of Mathematics, Natal University College, Pietermaritzburg, Natal (proposed by R. T. A. Innes);

Roland Dale Thompson, M.A., M.Sc., Lecturer in Mathematics and Physics, The University, Perth, Western Australia (proposed by A. D. Ross);

William Samuel Warren, 11 East Grave, Cardiff (proposed by D. J. Jones); and

Arthur Harry Witney, 5 Albert Road, Brockley, London, S.E. 4 (proposed by W. M. Witchell).

One hundred and seventy-eight presents were announced as having been received since the last meeting, including, amongst others :—

Louis Bell, The Telescope.
Cape Astrographic Zones, vol. v., zone $-45°$.
Catania, Reale Osservatorio, Astrographic Catalogue, vol. ii., part 1.
Dorpat Observatory, tome 25, cahier 1.
H. Spencer Jones, General Astronomy.
San Fernando Astrographic Catalogue, tomo 3, zone $-4°$.
Toulouse Observatory, Annales, tome 9, parts 1, 2.
Mount Wilson Observatory, three photographs of model of the 50-foot Michelson Interferometer Telescope (presented by Prof. Hale).

The President read the following resolutions of the Council :—

"That the Council desire to record their sense of the loss which the Society itself and Science in general have sustained by the death of Colonel E. H. Grove-Hills, who rendered important services to the Society as a Fellow since 1893, President 1913–15, Treasurer 1904–12 and at the time of his death.

"In requesting the President to convey their sympathy to Mrs. Grove-Hills in her bereavement, the Council desire him to ask her to accept it as an expression of their personal sorrow, as well as of their esteem for him as a man of science."

"That the Secretaries be requested to communicate to the eldest son of the late Mr. W. H. Wesley the deep sympathy felt by the Council in the bereavement which he and the other members of his family have experienced, and at the same time to express the high appreciation of the Council of the devoted and zealous services of their late Assistant Secretary in the discharge of his duties during the long period of forty-seven years.

"But it should be added that this faithful service was not by any means Mr. Wesley's whole claim on our gratitude. He rendered important services to astronomy by his skill as a draughtsman, of which the Society has a keen appreciation. And finally, it is felt by all who knew him that they have lost a personal friend, who was always ready with a genial welcome to the Fellows of the Society and all those whom they brought to Burlington House."

The W. Struve (Σ) *Double Stars.* By J. Jackson, M.A., B.Sc.

(*Communicated by the Astronomer Royal.*)

1. *Historical.*—The first systematic survey of the sky for double stars was made by F. G. W. Struve with the 9·6-inch refractor (focal length about 14 feet 4 inches) of the Dorpat Observatory. With this telescope the brighter stars, about 120,000 in number, between the North Pole and declination $-15°$ were examined between 1824 November and 1827 February. In this hurried examination, at the rate of about 400 an hour, many double stars actually within the power of the telescope were of course missed, but nevertheless as many as 3112 double stars, including several hundred double stars previously known, were recorded. The maximum separation allowed was 32″. Later 22 stars were added, but 494 were rejected for various reasons, leaving in all 2640 stars. Over 10,000 measures of these stars were made by Struve in the years 1828 to 1835 and published in the *Mensuræ Micrometricæ* in 1837.

These measures appear to be of extraordinary accuracy for the epoch in which they were made, and in double-star astronomy hold a similar place to that held by Bradley's observations in meridian astronomy. Many measures of the stars have been made in the ninety years that have elapsed since the first measures (although a number of stars within 30° of the pole have not been well observed since 1870), and a discussion of these measures may enable us to form an estimate of what may be expected from the observation of double stars.

2. *General Introduction.*—The period of revolution of visual binary stars is so long that in the majority of such systems no relative motion will be revealed in a century. This is in accordance with the known masses of binary stars deduced from systems of short period and relatively large parallax. From these systems it appears that the mass is subject to relatively small fluctuations, the average being about twice that of the Sun. Now the period for a system of this mass and separation 5″ is 7900 years for a parallax of 0″·01, or 22,400 years for a parallax of 0″·005. In ninety years therefore the total motion to be expected for these two typical systems is of the order of 4°·1 and 1°·4 in angle, or 0″·36 and 0″·13 in arc respectively. The probable error in the observations is of the order of these quantities, so that orbital motion in the Σ stars can only be expected to be shown when the separation is less than 5″ or the parallax larger than 0″·01.

Observations extending over an interval of ninety years should, however, reveal most of the optical systems. For example, a star with a parallax of only 0″·005 will have a parallactic motion of 2″ a century. It will therefore generally be possible to separate the Σ stars into physical and optical systems if any relative motion is shown. For physical systems the components will be close and relatively bright, while their relative motion will be small. When the separation is not so small and the relative motion ten times as large, the system may be considered as merely optical. Relatively few systems will be inter-

mediate in character, and these will be physical only if they have a large parallax.

For only about 650 of the Σ double stars has there been any appreciable relative motion, the others have been generally classified as "fixed." It is probable that the great majority of these "fixed" double stars form real physical systems in which the linear separation is too great for orbital motion to have been observed. The physical connection of the components of such systems is proved when they are found to have a common proper motion, When the proper motion is too small to exceed its probable error several times, no certain conclusion can be drawn. On account of the uncertainty of many of the proper motions, the double stars for which no motion has been observed have been classified according to the separation, and the magnitude of the brighter component. If, however, the micrometric observations of a double star are sufficiently numerous and accurate to prove the relative motion has not been greater than, say, 0″·2, and if the stars are not too faint, there can be little doubt that they form a physical system.

3. *Classification.*—In the following discussion Σ453, Σ1143 have been rejected as single. Twenty-one triple systems have been entered twice, but the other triple systems have been only entered once, as one of the three components is of little significance. For 9 of these 21 systems all three components show relative motion, while for the other 12 there is a moving and a "fixed" pair. For a number of the Σ stars one or other of the components has been found to be a close double by a later observer, but no consideration has been taken of double stars not actually observed by Struve. In all 2659 combinations have been treated.

The stars have been classified as follows:—

A. Pairs showing relative motion:
- I. Binary systems, 449.
- II. Optical systems, 161.
- III. Nature uncertain, 39.

B. Pairs showing no certain relative motion:
- IV. Close pairs, 1174.
- V. Intermediate pairs, 481.
- VI. Wide pairs, 355.

The stars in Class III. are discussed later, and it is concluded that very probably at least 27 are optical and only 12 are possibly binary.

The separation into Classes IV.–VI. has been made as follows, according to the magnitude of the brighter component as given in Lewis's memoir on the Struve stars (*Memoirs, Royal Astronomical Society*, 56). For Class IV. the maximum separation in seconds of arc was given as a function of the magnitude by the formula

$$\log \text{separation} = \log 5 + 0{\cdot}2(9{\cdot}0 - m).$$

This means that when the brighter component was of magnitude 9·0 the maximum separation allowed was 5″, and that for brighter stars a larger separation was allowed in such a way that if the system were

removed to the distance where the brightness would be reduced to magnitude 9·0, the separation would be reduced to 5″. The following table indicates the maximum separation in Class IV. for different magnitudes:—

Magnitude	9·0	8·5	8·0	7·5	7·0	6·5	6·0	5·5	5·0
Separation	5″·00	6″·30	7″·92	9″·98	12″·56	15″·81	19″·91	25″·06	31″·55

In Class V. twice this separation was allowed. Still wider pairs were included in Class VI.

In this classification we have probably given undue importance to the brighter stars. As, however, it has been our aim to leave out no systems of possible future importance, it has seemed advisable to follow this definite scheme. It is probable, as we have already seen, that nearly all of the "fixed" stars form physical systems. Observations of stars in Class V. and still more in Class VI. can hardly be expected to yield results of value for another century at least.

4. *Binary Stars.*—449 stars have been classified as binaries showing relative motion. A complete list of the Struve numbers of these stars is given in Table I. at the end of this paper. Of the 39 doubtful stars it is found later that only 12 are at all likely to be binary. These are not included in the following discussion.

In a paper on "The Hypothetical Parallaxes of 556 Visual Double Stars" (*Monthly Notices*, 81, 2), Mr. Furner and the writer have considered all the double stars observed at Greenwich Observatory in the period 1893–1919. In that paper 322 Struve stars are included, and it is unnecessary to repeat here the conclusions drawn from them. Similar results for the remaining 127 Struve stars are given in Table II. of the present paper. It is thus possible, by reference to these two papers, to obtain data for all the Struve binaries in motion. The actual relative motion from which the hypothetical parallaxes are calculated has been given for the earlier paper in the Greenwich volume on Double Stars recently published. The first column of Table II. gives the number of the star in Burnham's General Catalogue of Double Stars. The second and third columns give the spectral type and photometric magnitude according to the Henry Draper Catalogue. For the first 19 hours of Right Ascension the data have been taken from the published volumes of that catalogue. We have here to express our thanks to Dr. Shapley and Miss Cannon for kindly supplying unpublished material for the remaining 5 hours of Right Ascension. The fourth column gives the absolute magnitude corresponding to a parallax of 0″·1, calculated from the apparent magnitude and the hypothetical parallax given in the fifth column. The hypothetical parallax has generally been calculated from the expression $0{\cdot}397\sqrt[3]{\rho v^2}$, where ρ is the mean separation and v the annual motion in seconds of arc. In a few cases for which the second differential coefficient of the distance with respect to the time could be calculated, formula (5) of *Monthly Notices*, 81, 6, has been used. The actual motion from which the hypothetical parallaxes have been calculated is given in the sixth to the ninth columns of Table II. These give the relative positions of the stars for 1830 and 1910 deduced by

plotting both position-angle and separation against the time. The letters N, S, T in the last column have the following significance with reference to the notes following the table :—

N : Note to explain the motion more completely.

S : Spectroscopic parallax determined by Adams and his collaborators (*Astrophysical Journal*, 53, 13, or *Mount Wilson Contributions*, 199).

T : Trigonometric parallax. This has been changed from relative to absolute by addition of 0″·005. The number in brackets gives the number of determinations.

It has not been considered necessary to subject the new material to a discussion similar to that given in the paper by Mr. Furner and the writer. It is clear from an inspection of the table that the previous results as to the connection between spectral type and absolute magnitude are confirmed. It will be seen that there are several parallaxes exceeding 0″·05, and that in all cases these refer to dwarf systems.

At the end of Table II. is given the list of 12 stars for which a true binary character is by no means certain. The deduced parallaxes for these stars range from 0″·043 to 0″·085. Parallax determinations of these should be made by the trigonometric method.

It is of some interest to consider the distribution of moving binaries amongst the spectral types. For comparison the following table gives the number of Σ stars of various ranges of spectral type contained in the first 19 hours of Right Ascension of the Draper Catalogue and also the total number of stars (divided by 100) of the different spectral types in the Draper Catalogue (which extends approximately to the limit of magnitude of stars examined by Struve), as given by Dr. Shapley and Miss Cannon in *Harvard Circular*, No. 226 :—

Type.	Oe–B9.	A0–A5.	F0–F8.	G0–G5.	Later.
Σ Binaries	20	103	161	98	47
Σ Stars	166	490	563	353	238
All stars	164*	565	436	431	630
Σ Binaries / Σ Stars	0·12	0·21	0·29	0·28	0·20
Σ Binaries / All stars	0·12	0·18	0·37	0·23	0·07
Σ Stars / All stars	1·01	0·87	1·29	0·82	0·38

This table indicates that relatively there is no deficiency of double stars amongst the early spectral types, although the proportion of binaries in motion is small. In type F there is excess of double stars, and of these a relatively large proportion show relative motion. For the very late types there are few double stars, and of these a relatively small number show relative motion. It may on the whole be concluded that the proportion of double stars and of binaries is rather greater amongst dwarfs than amongst giants.

The number of Σ binaries arranged according to hypothetical parallax and spectral type is shown by the following table. A few stars for

* This includes types B0 to B9 only.

which the spectral types of both components are given in the Draper Catalogue are included twice :—

Hyp. Par. greater than	Oe–B9.	A0–A5.	F0–G0.	G5–M.	Total.	Percentage earlier than F0.
″ 0·099	0	0	4	5	10	0
·079	0	1	7	8	16	6
·059	0	1	15	13	29	3
·039	0	3	30	31	64	5
·029	0	11	48	48	107	10
·024	0	19	67	56	142	12
·019	0	29	99	65	193	14
·017	3	36	111	69	219	18
·015	5	43	129	73	250	19
·013	5	58	143	77	283	22
·011	11	61	167	78	317	22
·009	12	69	184	86	351	22
·007	16	88	196	90	390	26
Remainder	4	15	12	6	38	53

This table shows the real scarcity of early-type stars in space. It is only their extreme brilliance, enabling them to be seen at enormous distances, which causes them to appear in fair numbers in star catalogues. A typical giant star of absolute magnitude 1·0 has an apparent magnitude of 9·3 at a distance of 457 parsecs, while a dwarf of absolute magnitude 6·0 will appear of this apparent magnitude at a distance only one-tenth as great. If therefore we consider stars of the same apparent magnitude for giants and dwarfs, the former are scattered through a volume 1000 times as great as the latter. The last column of the above table indicates how the faint stars of late type are gradually lost as we pass to greater and greater distances. The following table gives an indication of the relative numbers of the different types in space by considering the different types within certain limiting distances so chosen that the loss through apparent faintness is not excessive :—

Type.	Limiting Parallax.	No. of Stars.	No. of Stars per Million Cubic Parsecs.
Oe–B9	″ 0·010	12	5
A0–A5	0·010	69	26
F0–G0	0·025	67	397
G5–M	0·040	31	752

This table, of course, refers only to Struve binaries showing motion. In the following tables the binaries have been tabulated according to

the separation as measured by Struve. The table also gives a comparison with the total number of Struve stars :—

	0″–1″.	1″–2″.	2″–4″.	4″–8″.	8″–16″	16″–32″.
Binaries showing motion	67	158	135	61	15	3
Struve stars *	86	317	535	582	582	535
Per cent. binaries	78	50	25	10	3	1

The closest pairs which have been classed as optical are 3 with separation 2″–4″ and 10 of separation 4″–8″. In addition there are 9 systems of separation 4″–8″ for which the observed relative motion may or may not be orbital.

The above enables us to form an estimate as to what additional measures of the Struve stars may be expected to reveal during the present century, *i.e.* by the time the interval covered by observations is doubled. The angular motion is reduced to 0·35 when the separation is doubled, but the total motion measured in arc is only reduced by the factor 0·71. We may therefore expect the motion to be discovered in about double the time. Hence by the end of this century we may expect the percentage of stars in each of the above classes for which orbital motion may be determined to be that indicated at present by a class with half the separation. By the end of this century we may therefore expect the number of binaries showing relative motion in each of the above classes to be about 77 (say), 247, 266, 147, 61, 14. The latter figures are rather uncertain on account of liability to miss small motions in wider pairs as well as through scarcity of observations in many cases. We may, however, expect 300 to 350 additional binaries to show orbital motion by the end of this century.

By means of the above table we are also able to form an estimate of the completeness of our lists of hypothetical parallaxes for varying degrees of separation. The following table gives hypothetical parallaxes for various separations on the assumption of the centennial angular motion shown :—

	1″.	2″.	4″.	8″.	16″.	32″.
Angular motion	10°	8°	5°	4°	3°	3°
Hypothetical parallax	0″·003	″·006	″·008	″·015	″·026	″·037

It may therefore be assumed that we have determined most of the parallaxes less than 0″·01 for all the 938 Σ stars with separation under 4″, but that for the wider pairs only the relatively large parallaxes have been found. This is shown by the average parallaxes determined in each of the classes :—

	0″–1″.	1″–2″.	2″–4″.	4″–8″.	8″–16″.	16″–32″.
No. of binaries	67	158	135	61	15	3
Average parallax	″·0167	″·0171	″·0248	″·0395	″·096	″·118

5. *Optical Double Stars.*—A double star may be formed by two stars lying in approximately the same direction from the solar system

* Struve gives 91 stars between 0″ and 1″. One star was counted twice ; 3 are here classed as 1″ – 2″ ; 1 is rejected as single.

but at quite different distances from it. Such double stars are only optical combinations, and are of no importance except for the determination of the relative proper motions and parallaxes of the components. But as they may occur in all lists of double stars it is of interest to calculate their probable numbers. This can be easily done when the number of stars of different apparent magnitudes is known. Let us suppose that the number of stars as bright as magnitude m is n, and the number as bright as the fainter magnitude m' is n'. Then the probable number of optical pairs with components as bright as m, m' and with separation not greater than a'' is

$$\left(nn' - \frac{n^2}{2}\right)\frac{\pi^2 a^2}{4 \cdot 180 \cdot 180 \cdot 3600 \cdot 3600}.$$

According to the Harvard determinations we have—

$$\text{for } m = 9, \quad n = 134{,}000$$
$$m' = 10, \quad n' = 373{,}000.$$

Hence the probable number of optical pairs with separation not exceeding 32″ (Struve's limit) and with the components at least as bright as the 9th and 10th magnitudes is 247 for the whole sky.

It remains to correlate the above figures with the actual observations. Struve examined about 120,000 stars between the North Pole and declination − 15°. This area is about 0·629 of the whole sky, and will include about 84,000 stars to magnitude 9·0 on the Harvard scale. Struve must therefore have examined stars on the average to about magnitude 9·3. As we shall see later, the limiting magnitude of the companion depended to some extent on the separation, but it probably reached on the average to about magnitude 10·0. Assuming limiting magnitudes of 9·3 and 10·0, we may expect to find amongst the Struve stars 201 optical pairs.

The above figures have been computed on the assumption of a uniform random distribution of the stars. The Milky Way is evidence that this is not strictly accurate. If we assume that three-quarters of the stars are distributed at random over half the sky, and the other quarter over the other half, the above numbers have to be multiplied by $1\frac{1}{4}$. As a more general distribution we may assume the sky divided into 10 equal parts containing stars in the ratio 1 : 2 : 3 : 4 : 5 : 6 : 7 : 8 : 9 : 10. The number of optical pairs in this distribution would be 1·27 times that for a uniform distribution.

In the preliminary classification we found 161 optical pairs. Of the 39 doubtful cases we conclude later that at least 27 are optical. For 24 stars, however, the separation exceeded the limit of 32″, so that we have to deal in all with 164 optical pairs within the proposed limit. A comparison of the theoretical number to different limiting separations will be of assistance to explain the degree of completeness with which the optical pairs have been found. We have already computed that

there should be 201 optical systems amongst the Struve double stars if the limiting magnitude of the companion was 10·0. The following table gives the required data for different limiting separations:—

Separation under	4″	8″	12″	16″	20″	24″	28″	32″
Theoretical number	3	13	28	50	79	113	154	201
Number found	3	15	37	67	91	121	145	164

From this table it appears that for separation under 16″ the limiting magnitude for the companion was fainter than 10·0. For separation over 16″ the number of pairs found is considerably below the calculated number. This may be in part due to some being missed in our examination, especially as the measures for such pairs are sometimes neither numerous nor very accurate, but it is probably due in larger measure to Struve rejecting many pairs of separation 16″ to 32″ on account of the faintness of the companion. It may fairly be concluded that nearly all the optical pairs with separation under 16″ have been found.

Table III. gives a list of the optical systems. Of the 39 doubtful systems, 11 concluded to be optical are included in Table III., while 16 probably optical are added at the end of the table. The columns of Table III. give the Struve and Burnham General Catalogue numbers, the magnitudes and separations taken from Burnham, vol. i.

6. *Pairs showing Relative Motion for which the Nature of the Motion is uncertain.*—In the preliminary examination of the stars for which there was fairly large relative motion between the two components, there were 39 pairs for which it was uncertain at the first inspection whether the motion was orbital or merely due to difference in proper motion. There was considerable probability that most of these pairs were optical, but it was felt undesirable to classify a combination as optical when it might turn out to be a real binary with a large parallax. For 37 of these stars the relative motion is between 1″·0 and 5″·0 per century, for the other 2 it is 5″·5 and 11″·2. These stars are treated in detail below. A value of the hypothetical parallax on the assumption that the motion is orbital is given. It will be seen that a direct determination of the parallax will at once settle the question whether or not these are binary stars. The magnitudes are taken from Burnham's General Catalogue, vol. i.

Of the 39 stars it is concluded that 11 are optical pairs, that 17 are probably optical, but that there is no real evidence as to the remaining 11, which may possibly be binary.

Σ 49. *β*.G.C. 368. Mags. 6·5, 10·0.—The separation has increased from 4″·46 in 1830 to 6″·80 in 1910, while the position-angle has remained practically unchanged at 320°·4. The hypothetical parallax is 0″·067. Spec. Go. Mag. 7·05. Abs. Mag. 6·2. The pair is very close to be merely optical, while the hypothetical parallax is not too great to be ruled out. The absolute magnitude is not improbable for a Go star. This is possibly a real binary.

Σ 434. *β*.G.C. 1827. Mags. 7·0, 7·8.—The motion is shown by

1830	88°·3	28″·30
1910	86·0	31·00

Assuming orbital motion, the hypothetical parallax is 0″·140. Spec. K5, A5. Mags. 7·40, 8·7. The value of the hypothetical parallax gives absolute magnitudes of 8·1, 9·4. In this case the hypothetical parallax is improbably great, while a star of absolute magnitude 9·4 and spectral type A5 is unusual. This system is very probably optical.

Σ 547. *β*.G.C. 2198. Mags. 8·5, 11·5.—The motion deduced from the observations without adjustment is shown by

1830	343°·8	4″·35
1850	351·8	3·23
1870	4·8	2·51
1890	26·7	2·04
1910	64·0	1·85

These positions lie very nearly on a straight line, and show relative motion of 4″·43 in 138°·7 for the 80 years. If we assume that the motion is orbital the hypothetical parallax is 0″·078. The star is not given in the Draper Catalogue. The motion certainly appears to be rectilinear, but the stars are very close to be only an optical combination. Modern observations are required for the determination of proper motion.

Σ 596. *β*.G.C. 2351. Mags. 8·0, 10·2.—The relative motion is shown by

1830	280°·7	11″·12
1910	290·0	10·12

On the assumption of orbital motion the hypothetical parallax is 0″·073, but the system is probably optical. If not, the close pair discovered by Hussey will be in rapid orbital motion.

Σ 613. *β*.G.C. 2430. Mags. 7·7, 8·7.—The relative motion is given by

1830	106°·7	19″·90
1910	103·8	16·06

If this were orbital motion the hypothetical parallax would be 0″·141. The Draper Catalogue gives Spec. Fo. Mag. 8·5. From this we deduce absolute magnitude 9·2. The system is therefore optical.

Σ 649. *β*.G.C. 2546. Mags. 7·0, 8·7.—The position-angle has decreased from 81°·2 in 1830 to 75°·1 in 1910. The separation has been stationary about 21″·44. The assumption of orbital motion gives

a hypothetical parallax of 0″·108. Spec. B8. Mag. 5·67. Abs. Mag. 5·8. Boss gives a P.M. for the brighter star of 0ˢ·0000, −0″·014. This explains about half the observed relative motion. This is an optical pair.

Σ 809. β.G.C. 3015. Mags. 7·7, 8·8.—The relative motion is given by

1830	101°·3	25″·67
1910	97·3	24·87

This would indicate a hypothetical parallax of 0″·103. Spec. G5, Fo. Mags. 7·9, 8·9. Abs. Mags. 8·0, 9·0. If the parallax were of the order of 0″·1 the close pair β1188 would be in rapid orbital motion. An optical pair.

Σ 829. β.G.C. 3096. Mags. 9·0, 10·7.—The motion in angle is rather uncertain.

1830	237°·9	16″·45
1910	234·5	18·07

This gives a hypothetical parallax of 0″·088. Spec. F8. Mag. 9·1. Abs. Mag. 8·8. The system is probably optical.

Σ 830. β.G.C. 3113. Mags. 8·2, 8·7.—The motion is given by

1830	249°·6	12″·83
1910	256·8	12·01

These figures give for the hypothetical parallax 0″·070. Spec. F2. Mag. 8·8. Abs. Mag. 8·0. Probably optical.

Σ 846. β.G.C. 3164. Mags. 8·2, 10·7.—The motion is rather uncertain.

1830	138°·3	12″·57
1910	136·2	13·09

These give a hypothetical parallax of 0″·042 ±. Spec. A5. Mag. 8·5. Abs. Mag. 6·6. Probably optical.

Σ 876. β.G.C. 3250. Mags. 8·5, 11·0.—There are not many observations, but they indicate motion as follows—

1830	51°·7	7″·66
1910	45·8	8·20

This would give a hypothetical parallax of 0″·044 ± if the motion were orbital. The system is probably only optical.

Σ 1025. β.G.C. 3858. Mags. 7·5, 7·8.—The relative motion is shown by

1830	141°·4	22″·58
1910	134·3	24·32

These positions indicate a hypothetical parallax of 0″·146. Spec. Ko. Mag. 7·7. Abs. Mag. 8·5. Probably optical.

Σ 1046. β.G.C. 3913. Mags. 8·6, 11·7.—The motion is not very certain.

1830	231°·0	11″·85
1910	236·7	10·60

If the system were binary this would give a hypothetical parallax of 0″·066. Spec. A2. Mag. 9·0. Abs. Mag. 8·1. Probably optical.

Σ 1047. β.G.C. 3909. Mags. 7·3, 9·8.—The motion is given by

1830	19°·5	20″·70
1910	24·8	22·33

The hypothetical parallax given by these figures is 0″·108. Spec. F2. Mag. 8·3. Abs. Mag. 8·5. The system is optical.

Σ 1091. β.G.C. 4062. Mags. 8·2, 8·7.—The relative motion is shown by

1830	335°·5	28″·58
1910	329·8	28·97

The resulting hypothetical parallax is 0″·143. System probably optical.

Σ 1135. β.G.C. 4249. Mags. 4·9, 11·0.—The position-angle has remained nearly constant about 211°, while the separation, as shown by the few observations available, has decreased from 22″·73 to 21″·11 between 1830 and 1910. This gives a hypothetical parallax of 0″·082. The spectroscopic parallax determined at Mount Wilson is only 0″·010, and Abetti has found a trigonometric parallax of 0″·028. The motion of A relative to B is −″·010, −″·017 annually, while the proper motion of A is −″·014, −″·006 according to Auwers, or +″·004, −″·040 according to Boss. The system is therefore merely optical. Spec. K2. Mag. 5·29.

Σ 1678. β.G.C. 6263. Mags. 6·3, 7·0.—The mean separation has been about 32″·39 during the period covered by the observations, but it probably reached a minimum of 32″·2 about 1870 and rose to about 32″·6 about 1910. The position-angle has decreased from 212°·2 in 1830 to 192°·3 in 1910, so that the relative motion is very large. The hypothetical parallax is 0″·341. Spec. A0. Mag. 6·76. Abs. Mag. 9·4. The system is optical.

Σ 1830. β.G.C. 6801. Mags. 8·5, 9·8.—The relative motion is large, as shown by

1830	263°·5	4″·81
1910	295·0	6·57

This gives a hypothetical parallax of 0″·090. Spec. G5. Mag. 9·4. Abs. Mag. 9·2. The system is very close to be only optical, but the motion of A relative to both B and C indicates that it is due to the proper motion of A. Probably optical.

Σ 2010. β.G.C. 7514. Mags. 5·0, 6·0.—The relative motion is well determined.

1830	9·6	31″·28
1910	10·5	29·38

This gives a hypothetical parallax of 0″·106. Spec. G5, G5. Mags. 6·52, 5·34. Abs. Mags. 6·7, 5·5. The spectroscopic parallax has been found to be ″·008 for each component, while the trigonometric parallax of the brighter star has been found at Yerkes to be −0″·011. The combination is therefore optical.

Σ 2017. β.G.C. 7542. Mags. 7·7, 8·4.—The relative motion is given by

1830	249°·6	25″·00
1910	253·4	26·90

This gives a hypothetical parallax of 0″·122. Spec. K2, K2. Mags. 8·1, 8·7. Abs. Mags. 8·5, 9·1. This is possibly an optical pair.

Σ 2080. β.G.C. 7708. Mags. 8·0, 11·8.—The motion is indicated by

1830	29°·3	5″·62
1910	24·0	2·86

The hypothetical parallax is 0″·068. Possibly binary.

Σ 2082. β.G.C. 7714. Mags. 4·0, 10·7.—The relative motion is shown by

1830	92°·4	22″·00
1910	92·0	24·23

This gives a hypothetical parallax of 0″·104. Spec. Ma. Mag. 5·14. Abs. Mag. 5·2. The spectroscopic parallax is 0″·008. The system is therefore optical.

Σ 2096. β.G.C. 7758. Mags. 6·0, 9·3.—The observations of distance are not very accordant.

1830	92°·7	21″·90
1910	90·5	22·70

These positions give a hypothetical parallax of 0″·063. Spec. A2. Mag. 6·04. Abs. Mag. 5·0. The system is probably optical.

Σ 2165. β.G.C. 8017. Mags. 7·0, 8·5.—The motion is given by

1830	45°·5	6″·67
1910	55·2	8·20

These positions give a hypothetical parallax of 0″·066. Spec. Fo. Mag. 7·61. Abs. Mag. 6·7. This is possibly a binary.

Σ 2169. β.G.C. 8032. Mags. 8·0, 10·0.—The separation has remained nearly constant at 14″·83, while the position-angle has increased from 88°·7 to 98°·4. Hyp. Par. 0″·100. Spec. F8. Mag. 8·6. Abs. Mag. 8·6. Probably optical.

Σ 2184. β.G.C. 8065. Mags. 6·3, 11·2.—The relative motion is given by

1830	76°·5	21″·43
1910	71·6	21·77

These positions give a hypothetical parallax of 0″·084. Spec. G5. Mag. 6·66. Abs. Mag. 6·3. This is probably optical.

Σ 2306. β.G.C. 8502. Mags. 7·2, 7·9.—The position-angle has remained nearly constant at 220°·1, while the separation has decreased from 12″·85 in 1830 to 11″·59 in 1910. This motion gives a hypothetical parallax of 0″·057. Spec. F5. Mag. 7·35. Abs. Mag. 6·1. The fainter star forms Δ 18, and as its components show no relative motion the parallax cannot be large. Hence Σ 2306 is an optical pair.

Σ 2364. β.G.C. 8721. Mags. 8·0, 10·2.—The relative motion is indicated by

1830	182°·3	6″·48
1910	177·4	7·95

This gives a hypothetical parallax of 0″·056. Spec. Ko. Mag. 8·0. Abs. Mag. 6·7. This is possibly a binary.

Σ 2441. β.G.C. 9000. Mags. 7·7, 9·3.—The position-angle has decreased from 290°·5 in 1830 to 276°·7 in 1910, while the separation has been nearly constant at 5″·33. These figures indicate a hypothetical parallax of 0″·043. Spec. A3. Mag. 8·2. Abs. Mag. 6·4. This is probably a binary.

Σ 2453. β.G.C. 9035. Mags. 8·2, 10·7.—The motion is given by

1830	99°·8	15″·06
1910	94·1	14·10

The hypothetical parallax is 0″·072. Spec. A2. Mag. 8·37. Abs. Mag. 7·7. This is probably an optical pair.

Σ 2564. β.G.C. **9521**. Mags. 8·5, 10·2.—The relative motion is well determined.

1830	185°·0	10″·70
1910	169·4	9·63

The hypothetical parallax is 0″·092. Spec. G5. Mag. 8·7. Abs. Mag. 8·5. Possibly optical.

Σ 2688. β.G.C. **10275**. Mags. 8·7, 9·8.—The position-angle has been nearly constant, while the separation has increased from 5″·56 in 1830 to 6″·63 in 1910. This gives a hypothetical parallax of 0″·041. The nature of this system is very doubtful.

Σ 2690. β.G.C. **10281**. Mags. 7·0, 7·2.—The position-angle has remained nearly constant about 256°·0, while the separation has increased from 14″·15 in 1830 to 15″·80 in 1910. This gives a hypothetical parallax of 0″·074. Spec. A0. Mag. 6·39. Abs. Mag. 5·7. The fainter star was resolved by Dawes, and as its components show no large relative motion, Σ 2690 is an optical pair.

Σ 2709. β.G.C. **10404**. Mags. 8·2, 10·0.—The position-angle has decreased from 314°·6 in 1830 to 307°·0 in 1910, while the separation has been constant about 9″·32. The hypothetical parallax is 0″·050. Spec. K0. Mag. 8·4. Abs. Mag. 6·9. Possibly binary.

Σ 2728. β.G.C. **10540**. Mags. 8·0, 10·3.—The position-angle has been nearly constant at 24°·4, while the separation has increased from 4″·00 to 6″·75 in the 80 years following 1830. This gives a hypothetical parallax of 0″·074. Spec. K5. Mag. 8·0. Abs. Mag. 7·3. Probably binary.

Σ 2824. β.G.C. **11222**. Mags. 3·9, 10·8.—The relative motion is well determined.

1830	309°·0	10″·98
1910	297·2	12·60

This gives a hypothetical parallax of 0″·098. Spec. F5. Mag. 4·27. Abs. Mag. 4·2. The spectroscopic parallax is 0″·030, and the mean of four trigonometric determinations is 0″·038. The hypothetical parallax for the close pair β 989 is 0″·046. The proper motion given by Auwers does not completely explain the relative motion.

Σ 2838. β.G.C. **11327**. Mags. 6·0, 8·8.—The relative motion is shown by

1830	185°·2	20″·65
1910	184·5	19·33

The hypothetical parallax is 0″·070. Spec. F8. Mag. 6·18. Abs. Mag. 5·4. This is probably an optical pair.

Σ 2840. *β*.G.C. **11323.** Mags. 6·0, 7·0.—The relative motion is small, but pretty well determined.

1830	194°·7	19″·92
1910	195·1	19·00

These positions give a hypothetical parallax of 0″·056. Spec. A. Mag. 7·26. Abs. Mag. 6·0. This is probably a binary.

Σ 3014. *β*.G.C. **12381.** Mags. 8·1, 10·4.—The position-angle has been nearly constant, while the separation has increased from 7″·05 in 1830 to 8″·07 in 1910. This change gives a hypothetical parallax of 0″·043. Spec. Go. Mag. 8·47. Abs. Mag. 6·6. This is possibly a binary.

7. *Stars with Separation under* 1″.—Struve placed 91 stars in this class. Of these one, Σ 1457, was counted twice, and one, Σ 453, has been rejected as single. In our discussion 70 of the remaining 89 have been found to show orbital motion. We have re-examined the remaining 19, and the results are given in the following table, which is in the same form as Table II. at the end. The motion is in all cases rather uncertain, and in some cases quite so. For many of these stars there are very few observations. The hypothetical parallax given must be considered as maximum values. The parallax for all these stars is considerably less than 0″·01; all the stars are giants.

β.G.C.	Spec.	Mag.	Abs. Mag.	Hyp. Par.	1830.	1910.	1830.	1910.
				″	°	°	″	″
672	A0	6·82	−1·1	·003	309·1	303·4	0·55	0·55
718	F5	6·45	+0·9	·008±	150·8	146·9	0·84	1·06
1114	F0	7·7	+0·6	·004±	270·6	266·3	0·60	0·60 N
1143	A0	7·47	<0	<·004	Change <7°		0·75	0·75
1169	A2	9·4	<4	<·010	...	...	0·80±	1·20±
1668	A0	7·9	<1±	<·004	Change <5°		0·93	0·93
1683	G5	7·35	1·0	·005	91·0	96·5	0·88	0·98
2748	A0	7·6	0·6	<·004	204·5	199·7	0·90	0·90
2758	G5	7·16	<0	<·004	Change <5°		1·00	1·00
3172	G0	8·4	<1	<·003	Change <4°		0·92	0·92
3557	B9 / B9	8·1	<1	<·004	Change <5°		0·91	0·91
3158	F8	7·7	<0	<·003	Change <5°		0·61	0·61
6810	G5	8·7	1·7	·004	124·5	134·3	0·59	0·59
6961	G5	7·77	<1	<·004	Change <5°		0·89	0·89
8844	A3	8·03	<2·8	<·009	...	...	0·75	1·03
8874	G5	7·8	0·2	·003±	32·6	29·0	0·97	0·97
10305	G?	6·34	0·9	·008±	75·6	79·3	0·86	1·09
11569	F?	6·76	−1·0	<·003	Change <4°		0·78	0·78
3549	B9	8·72	2·0±	·004±	93·3	90·6	0·61	0·73

β.G.C. 1114. Positions for 1830, 1870.—No observations since Dembowski. The determination depends on observations by Struve and Dembowski only.

8. *Burnham's 61-Cygni-Type Stars.*—In his general catalogue Burnham classified 38 stars as of the 61 Cygni type, *i.e.* double stars with both components in relatively rapid motion. Of these stars, 30 are Struve stars and 25 have here been classified as binaries. The remaining 5 are Σ 53, Σ 634, Σ 1329, Σ 1678, and Σ 3127. Σ 53 was rejected by Struve, while Σ 1678 has been discussed in § 6 as showing motion of an indeterminate kind, although probably optical. Σ 634, Σ 1329, and Σ 3127 were classified as optical, but a few remarks may be here made about them.

Σ 53. *β*.G.C. **384.** Mags. 8·0, 8·7.—This star was not measured by Struve, and was indeed not measured till 1891. The measures by Aitken in 1896 and Burnham in 1901 and 1907 show a relative motion of the brighter star relatively to the fainter of about 0″·29 in 215° annually. The following values of the proper motion have been found:—

Porter,	0″·415 in 216°·6
Küstner,	0·375 in 221·7
A. G. Nico,	0·301 in 228·8
Bauschinger,	0·453 in 223·9.

It appears that this is an optical system, and that the relative motion is largely due to the proper motion of the brighter star.

Σ 634. *β*.G.C. **2548.** Mags. 4·5, 7·9.—Burnham gives the P.M. of the brighter star as 0″·179 in the direction 323°·6, and from the relative motion determines the proper motion of the fainter star as 0″·165 in nearly the opposite direction. During the time covered by the observations the separation has decreased from 34″ to 12″, while the relative motion has been practically uniform. If this were orbital motion it would indicate a hypothetical parallax of 0″·55. All the indications, therefore, point to this being an optical system.

Σ 1329. *β*.G.C. **4999.** Mags. 8·3, 8·5.—Burnham gives the motion of the brighter star relatively to the fainter as 6″·108 in 216°·0. A. G. Nico gives the P.M. of the brighter star as only 0″·09 in 180°, but Furner and Storey have found it to be ″·110 in 223°. Hence the motion is explained by the proper motion of the brighter star.

Σ 3127. *β*.G.C. **7922.** Mags. 3·0, 8·1.—This star, *δ* Herculis, is one of the most difficult to classify in the whole heavens. The proper motion of the brighter star is well known, and, using the relative motion determined from micrometric observations, Boss gives the P.M.'s of the two stars as −0ˢ·0018, −0″·163 and −0ˢ·0077, −″·003. We thus have two stars with P.M.'s of the order of 16″ and 12″ a century now only about 12″ apart. The improbability of two such stars being unconnected is great, and in fact from these motions Struve asserted that the stars must be physically connected. However, the relative path has remained straight and the speed constant, while the separation has decreased to less than half its original value. If we assume the motion to be really orbital, the hypothetical parallax is about a third of a second.

There are two modern determinations of the trigonometric parallax with a mean of 0″·027. If this is a real binary system the mass will be 1000 times that of the sun. We therefore conclude that this is an optical system, though most unexpected.

9. *General Concluding Remarks.*—We have found amongst the Struve stars 440 binaries in orbital motion, 161 optical pairs in relative motion, and 39 moving systems for which the nature of the motion is uncertain. We have come to the conclusion that the majority of the optical pairs have already been detected, and that by the end of the present century it should be possible to find about 300 other binaries.

The question arises as to which stars will best repay observation. For a few systems in relatively rapid motion and for which it shall soon be possible to compute orbits, measures once every 5 or 10 years are desirable, and it may be every year during the description of critical parts of the orbits. For those binaries in which the relative motion has only been about 5° and for the "fixed pairs" it should not be necessary to repeat observations for 50 years, provided good observations have been secured since the beginning of this century. There are a number of pairs (particularly in high declination) which have not been observed for about 50 years.

The present analysis leads to the most hopeful prospects for the future of double-star astronomy. The following table, given by Aitken,* shows the number of close double stars at present known, with the names of some of the leading discoverers and the approximate epoch of their observations :—

	0″–1″.	1″–2″.
W. Struve (1828–1835)	90	314
Otto Struve (1843–1849)	154	63
Burnham (1870–1892)	385	305
Hussey (1899–1905)	674	310
Aitken (1899–1915)	1502	657
Total to mag. 9·0 B.D.	2206	1222

This table shows that an enormous advance in the discovery of close double stars has been made since Struve. It is true that the later stars are on the whole fainter. The last line of the table is only complete to magnitude 9·0 B.D. (about 10·0 on the Harvard scale), and there have been many recent discoveries of close pairs, especially by Jonckheere and Espin, between magnitudes 9·1 and 9·5 B.D. Many of the later discoveries are of very close double stars, and in fact rather more than half of those with separation under one second are actually under half a second. Within 80 years from the time of observation we may therefore expect some 2000 of the later discoveries to show sensible motion. One point, however, must be remembered. For a considerable proportion of the Otto Struve and Burnham stars the companion is very faint compared with the brighter star. This leads to a marked personality in the observations made by many observers, and there is no certain method of eliminating it. Consequently, a relatively

* *The Binary Stars*, pp. 26, 261.

large change must take place before it can be accepted as real. The stars measured by W. Struve are for the most part easy, and this may, partly at least, explain the remarkable accuracy of his measures. Aitken, indeed, has stated that the accuracy of observations with the 12-inch telescope at Lick is nearly as great as that with the 36-inch for stars of corresponding difficulty. If the components can be tolerably separated with the telescope, the actual aperture is of little importance.

In spite of the very short time which has elapsed since his earliest measures, Aitken has already detected motion in over 100 of his close doubles. The difference in epoch is 15 years or less. We may therefore hope that by the middle of the present century the number of binaries showing relative motion will have largely increased, especially as Aitken is engaged in re-measuring all the double stars discovered by himself and the closest ones discovered by Hussey. The attention of observers may be drawn to opportunities offered by the Burnham stars.

It will be seen that the best prospects are in the very close pairs. Unfortunately these are only observable with large telescopes under very favourable atmospheric conditions. There are, however, considerable numbers of double stars with separation of the order 1″–2″ for which observations are required.

In conclusion, it may be stated that the wider pairs might be observed photographically. Hertzsprung has shown that it is possible in nearly all cases to obtain good measurable photographs down to a separation of 5″, and that in favourable cases as to the relative magnitudes down even to 2″. The photographic method when applicable appears to be very accurate, and as plates can be preserved for long intervals, difficulties arising from personal peculiarities of the observer may be to some extent limited. It must, however, be remembered that photographic measures of double stars are not comparable with visual measures unless made through a yellow filter, as the difference in the refraction for different coloured stars depends on the wave-length of the light used. At present, however, no systematic observations of double stars are being carried out by the photographic method.

TABLE I.

Binaries showing Relative Motion (Struve Numbers are given).

2	148	202	285	346	460	535	589	676
13	149	208	295	360	461	536	609	677
42	158	226	296	367	483	544	615	712
52	162	227	300	380	493	546	616	716
60	174	228	305	395	498	554	619	728
61	178	234	310	400	511	562	622	736
67	183	248	312	408	516	565	635	742
73	185	257	314	412	518 AB	566	648	749
91	186	262 AB	326	422	518 BC	567	652	774
113	194	262 AC	333	425	520	572	657	787
138	196	278	334	443	531	577	670	795

TABLE I.—*continued.*

799	1175	1429	1711	1932	2135	2410	2606	2878
807	1177	1431	1714	1934	2140	2415	2627	2881
821	1187	1434	1716	1937	2153	2422	2628	2900
824	1194	1439	1722	1938	2161	2434 BC	2640	2909
826	1196 AB	1454	1728	1944	2163	2437	2642	2910
840 BC	1196 AC	1457	1734	1954	2170	2438	2652	2912
851	1202	1465	1752	1956	2171	2451	2653	2924
881	1205	1476	1757	1957	2173	2454	2658 AB	2928
888	1211	1487	1753	1965	2180	2455	2672	2934
905	1213	1500	1768	1967	2199	2461	2673	2944 AB
910	1216	1504	1769	1969	2203	2466	2720	2947
918	1223	1510	1772	1985	2205	2478	2723	2950
919 BC	1224	1517	1777	1988	2215	2479	2725	2958
932	1271	1523	1781	1989	2218	2481	2726	2979
936	1273	1527	1785	1998	2220	2482	2727	2987
945	1280	1534	1788	1999	2225	2484	2729	2990
948	1287	1536	1802	2006	2236	2486	2737 AB	3001
950	1291	1542	1808	2016	2244	2488	2737 AC	3006
963	1300	1543	1817	2021	2262	2491	2744	3017
981	1306	1555	1819	2022	2267	2509	2746	3033
982	1313	1561	1820	2023	2272	2513	2749	3041 BC
991	1316	1601	1825	2026	2275	2518	2751	3042
998	1321	1604	1834	2032	2281	2524	2752	3045
1009	1332	1606	1837	2049	2283	2525	2758	3046
1037	1333	1608	1863	2052	2289	2536	2762	3050
1051	1334	1613	1864	2054	2292	2541	2780	3056
1055	1338	1619	1865	2055	2294	2544	2783	3059
1066	1339	1621	1867	2078	2303	2545	2786	3060
1068	1348	1626	1871	2084	2315	2553	2799	3062
1074	1355	1639	1876	2094	2320	2556	2802	3063
1081	1356	1643	1877	2097	2323	2574	2804	3091
1093	1362	1647	1878	2101	2329	2576	2822	3105
1104	1374	1658	1879	2106	2367	2579	2825	3107
1110	1389	1661	1883	2107	2369	2580	2827	3114
1126	1400	1663	1888	2114	2375	2583	2837	3115
1146	1423	1670	1907	2117	2382	2596	2843	3121
1157	1424	1687	1909	2118	2383	2597	2862	3123
1165	1426	1695	1925	2119	2384	2603	2863	3125
1171	1428	1699	1926	2130	2398	2605	2872	

Stars showing relative motion, possibly binaries 49, 547, 2080, 2165, 2364, 2441, 2688, 2709, 2728, 2824, 2840, 3014.

TABLE II.

This table is additional to those given in *M.N.*, 81, 22–30, and gives the hypothetical parallaxes, etc., in the same form.

β.G.C.	Spec. Type.	Mag.	Abs. Mag.	Hyp. Par.	Position Angle. 1830.	Position Angle. 1910.	Separation. 1830.	Separation. 1910.	
				″	°	°	″	″	
10	Go	9·0	5·2	·017	233·0	215·7	1·86	1·86	
92	B9	6·23	1·8	·013	125·2	79·1	0·59	0·74	N
388	F5	7·52	3·0	·012	26·0	11·0	1·42	1·42	
439	Fo	6·29 6·29	1·8 1·8	·013	119·1	116·4	4·46	4·46	S, T
591	F5	7·04	3·5	·020	326·1	319·1	3·85	3·85	
903	G5	8·5	3·2	·009	130·0	141·0	1·28	1·28	
963	F5 A2	5·89	0·9	·010±	170·2	166·5	2·82	2·82	S, T
1027	Ao	6·64	1·7	·010	37·3	25·5	1·31	1·31	
1043	...	...	...	·011±	55·4	49·9	2·38	2·38	
1139	G5	7·61	2·6	·010	250·1	245·1	2·28	2·28	
1164	Go	8·08	3·8	·014	239·6	...	0·84	...	N
1262	A5p	4·59	1·6	·027 ·023	276·5 107·5	251·2 111·4	2·11 7·56	2·11 7·56	AB AC
1321	A2	8·02	1·3	·005±	82·3	75·6	0·54	0·54	N
1393	F8	4·22	4·2	·106	290·9	301·0	13·87	17·97	N, S, T
1457	Go	7·66	4·0	·019	13·6	22·2	3·41	3·41	
1490	G5	7·45	6·2	·056	217·0	217·0	9·13	7·65	T
1719	F8	8·7	3·8	·010	106·7	99·5	1·81	1·81	
1743	A3	7·55	2·4	·009±	345·5	335·1	1·38	1·38	
1747	F5	6·48	3·6	·026	282·6	353·0	1·53	0·38	N
1787	Go	6·12	5·1	·063	231·5	249·9	6·12	6·45	S, T
1854	G5	8·2	6·1	·038	44·3	48·0	9·08	8·50	S, T
1917	G5	8·2	2·5	·007±	104·2	114·2	1·15	1·15	
1952	F8 A2	5·25	1·6	·019	351·2	55·0	0·85	0·85	
2060	...	...	...	·009	174·0	165·5	1·35	1·35	
2088	Ao	6·91	2·4	·013	321·5	236·4	0·60	0·28	N
2102	Ko	5·13	2·6	·031	154·0	148·4	6·45	6·45	
2109	G5 A	4·48 9·7	6·3 11·6	·235	107·3	104·9	82·2	82·2	N
2157	A5	7·51	3·2	·014	152·0	165·0	1·76	1·76	
2169	Fo	7·26	1·7	·008	291·8	305·4	1·02	1·02	
2188	Go	8·3	4·7	·019	356·8	351·0	2·17	2·65	
2230	Fo	5·70	3·1	·030	...	...	...	...	N

TABLE II.—*continued.*

β-G.C.	Spec. Type.	Mag.	Abs. Mag.	Hyp. Par.	Position Angle. 1830.	Position Angle. 1910.	Separation. 1830.	Separation. 1910.	
				″	°	°	″	″	
2274	K0	7·25	1·7	·008	179·2	173·6	1·60	1·60	
2375	F8	8·19	5·2	·026	82·2	73·6	1·91	2·68	
2435	A0	4·99	2·5	·032	351·5	355·3	6·46	5·82	
2457	A0	8·0	3·7	·014	336·5	358·4	1·32	1·32	
2503	B9	7·86	0·9	·004±	280·6	291·7	0·63	0·63	N
2551	G5	7·9	4·5	·021	74·3	69·2	4·70	4·70	
2573	F5	6·07	−0·5	·005	184·4	182·1	1·76	1·76	
2652	F8	7·65	2·2	·008±	281·3	268·5	0·96	0·96	
2657	G0	7·27	4·4	·027	281·6	243·0	1·62	1·62	N
2926	A2	6·13	1·9	·014	36·1	19·3	1·43	1·43	
2945	F2	7·79	2·0	·007	78·7	72·1	1·28	1·28	
3017	F8	7·7	3·3	·013	139·7	148·2	2·26	2·26	
3127	F8	7·84	3·5	·013	217·8	204·7	1·53	1·53	S
3291	A3	7·16	3·1	·015	246·0	253·6	2·85	2·85	T
3382	G5	8·9	1·6	·006	170·7	158·0	0·77	0·77	
3948	A5	6·28	3·0	·022	344·3	328·2	2·30	2·30	
3962	F0	6·96	2·0	·010	267·5	282·0	1·30	1·30	
3990	B9	6·83	1·4	·008	120·0	147·0	0·64	0·64	
4269	F5	5·52	1·3	·014	17·6	12·8	3·35	3·35	
4365	A0	8·11	1·0	·004±	265·1	274·7	0·61	0·61	
4547	...	...	...	·046	327·8	318·7	7·18	7·18	
4815	K2	8·6	8·2	·085	33·5	45·8	7·50	4·67	N, S, T
4821	F0	7·8	3·8	·016	106·0	90·8	1·75	1·75	
4941	G0	8·4	6·2	·036±	144·5	137·9	6·81	6·81	
4944	G5	8·48	3·3	·009	240·3	258·5	0·94	0·94	
5003	F5	7·20	4·4	·028	16·4	22·8	5·75	5·75	
5032	A2	8·8	2·7	·006±	74·0	68·3	1·25	1·25	
5136	F0	6·43	3·9	·032	137·4	129·2	5·00	5·00	
5272	F5	7·9	4·8	·023	228·3	224·0	1·79	2·55	
5397	F5	7·9	2·8	·010	257·5	285·5	0·74	0·74	
5422	F5	7·36	3·3	·016±	85·0	86·8	3·77	3·44	
5501	K0	8·7	5·1	·019±	307·2	315·3	3·37	3·37	
5785	F0	6·86	4·0	·027	264·4	264·4	2·52	3·34	
6046	G0	6·78	5·9	·066	93·4	90·5	12·02	10·52	N
6064	K0	7·50	6·5	·063	223·9	222·5	10·55	12·05	N
6108	G0	8·2	2·6	·008	8·3	4·5	2·15	2·15	
6318	A2	5·84	2·3	·019	289·2	283·9	3·25	3·60	
6398	K0	7·7	3·7	·016	343·0	336·8	3·30	3·30	

TABLE II.—*continued.*

β.G.C.	Spec. Type.	Mag.	Abs. Mag.	Hyp. Par.	Position Angle. 1830.	Position Angle. 1910.	Separation. 1830.	Separation. 1910.	
				″	°	°	″	″	
6571	G5	7·82	4·6	·023	23·8	28·0	2·86	2·19	
6795	F5	6·36	4·1	·036	185·8	171·7	3·60	4·20	
6988	F2	6·17	3·7	·033	336·7	327·0	3·05	3·90	
7185	F8	7·14	1·5	·007	...	...	...	...	N
7191	Fop	7·09	3·2	·016	260·6	255·1	1·60	1·02	
7193	G5	7·95	5·9	·039	6·6	12·3	4·16	5·18	
7288	...	8·7	3·1	·008	272·6	268·0	2·21	2·21	
7317	G5	8·3	5·3	·025	39·9	39·9	2·74	1·90	
7323	F8	7·67	2·9	·011±	160·2	154·1	1·48	1·18	
7433	Go	6·72	5·1	·046	325·3	337·8	5·60	5·93	
7488	Ko	6·93	5·2	·045	102·9	100·3	10·45	11·27	T
7540	A3	8·3	5·5	·028	148·5	146·1	6·90	7·33	
7854	F8	7·35	3·3	·015	18·0	12·2	1·95	2·32	
8025	F2	7·81	2·6	·009	73·7	65·1	1·59	1·59	
8057	Fo	6·97	1·9	·010	265·6	262·5	3·18	3·18	
8162	G5	3·48	4·3	·147	241·7	244·7	29·92	32·54	N
	Mb	9·7	10·5						
8736	G5	6·88	2·7	·015±	...	...	...	...	N
8759	G5	8·1	4·8	·022	...	...	...	...	N, T
8783	A3	5·06	2·1	·026	25·9	11·2	3·11	3·11	N, T
		6·02	3·0						
8785	A5	5·14	3·0	·037	156·0	123·5	2·70	2·24	N, T
		5·37	3·2						
8798	K5	8·76	11·2	·302	133·5	151·8	12·22	17·16	N, S, T
9053	Fo	5·04	2·7	·035	330·8	313·4	3·66	3·66	T
9090	A3	7·31	4·8	·032	38·2	31·5	6·78	6·78	N
9136	F8	7·69	3·4	·014	221·5	228·8	2·49	2·49	S
9137	G5	6·84	5·9	0·67	224·4	217·6	10·58	9·30	S, T
		6·62	5·7						
9235	F5	6·91	2·9	·016	351·5	335·0	0·50	1·17	N
9434	Ko	8·5	6·8	·046	340·3	327·7	2·82	4·34	N
9464	A5	6·60	3·0	·019	315·3	322·0	3·57	3·57	
9617	F5	5·03	4·0	·061	73·4	70·8	25·75	25·64	N, S, T
9713	Ko	3·99	0·7	·022	355·0	366·0	2·79	3·11	S, T
9765	A3	4·80	−0·3	·009	184·6	181·2	3·40	3·40	
9935	A2	6·18	3·7	·032	27·6	19·4	4·90	5·40	
10044	F5	7·02	4·8	·036	126·8	116·3	5·42	5·42	S, T
10506	G8	4·34	2·0	·034	57·1	63·7	6·40	6·40	S, T

TABLE II.—*continued.*

β.G.C.	Spec. Type.	Mag.	Abs. Mag.	Hyp. Par.	Position Angle. 1830.	Position Angle. 1910.	Separation. 1830.	Separation. 1910.	
				″	°	°	″	″	
10509	G5	5·47	4·2	·057	273·7	269·8	11·91	10·83	S, T
10713	B9	5·74	−0·1	·007±	346·6	350·5	1·75	1·75	
10727	Ko	7·24	4·9	·035	145·5	154·3	5·51	5·51	
10832	B2	5·65	−0·9	·005	228·8	223·1	1·15	1·15	
10854	Ao	7·02	2·7	·014	43·6	25·0	1·33	1·02	
10896	A2	6·91	0·1	·004	183·6	186·5	2·50	2·50	
11245	A5	8·1	5·3	·028±	210·3	204·3	4·24	4·80	N
11267	G5	8·7	6·3	·034	322·5	291·8	2·15	2·65	
11334	A2 G	6·41	2·7	·018	133·1	141·3	2·35	1·95	
11483	A3 G	4·57 6·47	3·1 5·0	·050	289·5	281·6	5·65	7·01	S, T
11542	Ao Ao	6·56	1·1	·008	336·5	317·8	0·52	0·74	
11828	F2 A5	6·02	0·9	·009	257·5	274·5	0·86	0·66	T
11866	G5	8·1	6·4	·045	328·0	310·3	4·70	4·12	
11997	F5	6·39	3·9	·032	76·0	63·5	3·07	3·73	
12021	Go	5·80	1·5	·014	319·5	309·5	2·14	2·14	
12204	Go	7·22	3·6	·019	167·1	161·1	3·44	3·68	
12304	G5	4·90	2·7	·036	173·6	201 0	2·33	2·83	S, T
12392	Fo	7·17	2·7	·013	35·5	30·5	2·23	1·97	
12601	F5	7·05	4·3	·029	89·6	88·1	4·21	4·93	
12625	A2	8·2	2·7	·008±	260·0	266·0	1·66	1·66	
12639	G5	8·5	6·1	·034	232·2	251·2	2·52	3·22	
12731	Ko	7·15	0·1	·004	158·2	148·0	0·58	0·58	
12738	...	...	...	·011	335·0	328·6	2·24	2·24	

Possible Binaries.

β.G.C.	Spec. Type.	Mag.	Abs. Mag.	Hyp. Par.	Position Angle. 1830.	Position Angle. 1910.	Separation. 1830.	Separation. 1910.
				″	°	°	″	″
368	Go	7·05	6·2	·067	320·4	320·4	4·46	6·80
2198	...	8·5	8·0	·078	343·8	64·0	4·35	1·85
7708	...	8·0	7·2	·068	29·3	24·0	5·62	2·86
8017	Fo	7·61	6·7	·066	45·5	55·2	6·67	8·20
8721	Ko	8·0	6·7	·056	182·3	177·4	6·48	7·95
9000	A3	8·2	6·4	·043	290·5	276·7	5·33	5·33
10275	...	8·7	6·8	·041	173·1	173·1	5·56	6·63
10404	Ko	8·4	6·9	·050	314·6	307·0	9·32	9·32
10540	K5	8·0	7·3	·074	24·4	24·4	4·00	6·75
11222	F5	4·27	4·2	·098	309·0	297·2	10·98	12·60
11323	A	7·26	6·0	·056	194·7	195·1	19·92	19·00
12381	Go	8·47	6·6	·043	280·2	280·2	7·05	8·07

Notes.

β.G.C.
92.

1830	125°·2	0″·59
1850	111·4	0·65
1870	99·7	0·69
1890	89·0	0·72
1910	79·1	0·74

439. Spectroscopic parallaxes of 65^1 and 65^2 Pisc. 0″·008, 0″·011. Trig. par. 0″·010 (1). Hyp. par. 0″·013.

963. Spectroscopic parallax of brighter component 0″·010. Trig. par. 0″·017. Hyp. par. 0″·010.

1164. The motion till 1870 is shown by

1830	239°·6	0″·84
1870	222·2	0·64

A few observations have been made since 1900, but it is difficult to draw any definite conclusion from these. The rate of description of area deduced from the observations of 1830-1870 show that there would be a change in position-angle of 135° in 50 years for a mean separation of 0″·30. This star should be regularly observed with large instruments. Aitken has recently made observations on 3 nights.

1921·70	285°·9	0″·28

1321. The positions are for 1830 and 1870. There are no observations later than those of Dembowski.

1393. The positions are for 1780 and 1920. The spectroscopic parallax for θ Persei is 0″·087. Trig. par. 0″·083 (3). Hyp. par. 0″·106.

1490. The binary nature of the pair is assured by a large common proper motion. Trig. par. 0″·061. Hyp. par. 0″·056.

1747. The relative motion is shown more clearly by

1830	282°·6	1″·53
1850	287·9	1·25
1870	296·3	0·96
1890	312·8	0·67
1910	353·0	0·38

This star should be carefully followed.

1787. The spectroscopic parallax of the brighter star is 0″·069. Trig. par. 0″·029 (2). Hyp. par. 0″·063.

1854. The spectroscopic parallaxes of the brighter and fainter components are 0″·035 and 0″·032. Trig. par. 0″·030 (5). Hyp. par. 0″·038.

2088. The motion is more completely shown by

1830	321°·5	0″·60
1850	309·5	0·55
1870	295·3	0·50
1890	274·7	0·41
1900	260·5	0·35
1910	236·4	0·28

2109. This is a very rough determination from the stars AB. It is sufficient to prove that the three stars belong to one system. Spec. par. 0″·200. Trig. par. 0″·182. Hyp. par. BC 0″·119. Hyp. par. AB 0″·235.

2230. The hypothetical parallax is deduced from the orbit elements by Van den Bos in *M.N.*, **81**, 474.

2503. Unpublished observations by Aitken on 2 nights.

1918·75	293°·0	0″·72

2657. The separation appears to be decreasing and the motion in angle increasing.

3127. Spectroscopic parallax 0″·008. Hyp. par. 0″·013.

3291. Trig. par. 0″·006 (1). Hyp. par. 0″·015. Hyp. par. for close pair β895, 0″·009.

4815. The motion is indicated more closely by

1830	33°·5	7″·50
1870	38·8	6·30
1910	45·8	4·67

The spectroscopic parallaxes for the S and N stars are 0″·100 and 0″·105 respectively. Trig. par. 0″·086 (2). Hyp. par. 0″·085.

6046 and 6064. These pairs of stars are close together in space and probably physically connected. The stars are typical dwarfs.

7185. Hyp. par. deduced from a decrease in the separation from 0″·5 to 0″·2 in 70 years.

7488. Trig. par. 0″·046 (3). Hyp. par. 0″·045.

8162. From the orbit elements of the close pair A.C. 7 we deduce a hypothetical parallax of 0″·084. Spec. par. 0″·110. Trig. par. 0″·105 (5). Hyp. par. for wider pair 0″·147.

8736. The hypothetical parallax has been deduced from an assumed period of 80 years and a semi-major axis of 0″·34.

8759. The motion from 1830 to 1854 is shown by

1832	310°·3	0″·77
1843	317·5	0·60
1854	332·5	0·37

These positions deduced from the early observations are inconsistent with recent observations, which show the position-angle nearly constant about 308°, while the separation increased from 0″·40 in 1890 to 0″·69 in 1905. If we assume motion of 0″·40 in 22 years at a mean separation of 0″·60, the hypothetical parallax is found to be 0″·023; while if we assume a total motion of 0″·77 + 0″·69 in 73 years at a mean separation of 0″·365, the hypothetical parallax is 0″·021. Trig. par. 0″·004 (1).

8783 and 8785. These stars are ϵ^1 and ϵ^2 Lyræ. The hypothetical parallaxes 0″·026, 0″·037 agree very well. The trigonometric parallaxes 0″·000, 0″·014 (1) appear to be too small.

8798. The observed motion is given by

1830	133°·5	12″·22
1870	143·9	15·90
1910	151·8	17·16

This is a well-known dwarf star. Spectroscopic parallax 0″·240. Trig. par. 0″·289 (5). Hyp. par. 0″·302.

053. Trig. par. 0″·009 (1). Hyp. par. 0″·035.

9090. The brighter component is Δ 19. The motion is shown by

1860	44°·4	0″·73
1900	15·6	0·73

These positions give a hypothetical parallax of 0″·016.

9136. Spec. par. 0″·013. Hyp. par. 0″·014.

9137. The spectroscopic parallaxes of the brighter and the fainter components are 0″·066 and 0″·055 respectively. Trig. par. 0″·045 (5). Hyp. par. 0″·067.

9235. The motion is given in greater detail by

1830	351°·5	0″·50
1870	341·4	0·87
1910	335·0	1·17

9434. The motion is better indicated by

1830	340°·3	2″·82
1870	332·7	3·58
1910	327·7	4·34

9617. This pair is physically connected with β.G.C. 9602, for which a hypothetical parallax is given in *M.N.*, **81**, 29. The brighter and fainter components for 9617 have spectroscopic parallaxes of 0″·038, 0″·048 respectively. The parallax determinations for the two systems are as follows:—

	Spec.	Trig.	Hyp.
	″	″	″
9602	0·032	0·041 (1)	0·056
9617	0·043	0·045 (3)	0·061

9713. Spec. par. 0″·022. Trig. par. 0″·002 (1). Hyp. par. 0″·022.

10044. Spec. par. of brighter star 0″·017. Trig. par. 0″·016 (1). Hyp. par. 0″·036.

10506. Spec. par. 0″·024. Trig. par. 0″·014 (1). Hyp. par. 0″·034.

10509. The spectroscopic parallaxes of the brighter and fainter components of γ Delphini are 0″·021, 0″·032 respectively. Trig. par. 0″·026. Hyp. par. 0″·057.

11245. This determination depends on observations on 9 nights only.

11483. The spectroscopic parallax of the fainter star is 0″·030. Trig. par. 0″·040 (2). Hyp. par. 0″·050.

11828. Trig. par. 0″·010 (1). Hyp. par. 0″·009.

12304. Spec. par. 0″·014. Trig. par. 0″·032 (1). Hyp. par, 0″·036.

TABLE III.

Optical Systems.

Σ.	β.G.C.	Mags.	Sep.	Σ.	β.G.C.	Mags.	Sep.
			″				″
23	118	7·6, 9·9	12·70	80	519	7·8, 8·2	18·26
27	165	6·8, 10·7	31·67	86	560	8·0, 8·7	12·12
30	205	6·8, 8·7	31·23	112	706	8·5, 9·0	23·64
32	248	6·8, 10·6	13·67	117 A	732	4·5, 8·9	32·22
44	340	8·3, 9·0	7·86	118	759	8·5, 9·4	10·75
45	346	7·0, 10·0	8·79	125	761	7·9, 10·3	16·91
63	444	8·2, 11·2	11·42	132	794	7·0, 10·0	24·25
9	474	8·5, 9·7	21·44	142	860	8·2, 8·4	25·29

TABLE III.—*continued.*

Σ.	β.G.C.	Mags.	Sep.	Σ.	β.G.C.	Mags.	Sep.
			″				″
143	862	7·7, 9·0	30·31	1047	3909	7·3, 9·8	20·66
171	953	8·5, 8·5	27·89	1071	3991	8·2, 10·2	15·52
175	974	8·2, 8·5	10·43	1135	4249	4·9, 11·0	22·60
177	980	8·5, 9·0	34·27	1136	4265	7·3, 11·0	11·61
197	1050	7·3, 8·3	18·33	1142	4264	8·0, 10·4	24·36
254	1224	8·5, 10·0	13·33	1179	4418	8·5, 8·5	17·91
293	1389	8·5, 11·7	6·61	1230	4624	8·3, 10·0	28·00
325	1487	8·2, 9·7	11·70	1234	4645	7·0, 8·3	20·76
328	1492	8·5, 9·0	27·06	1236	4648	8·0, 8·5	35·79
335	1535	8·0, 8·5	24·38	1240	4660	7·2, 10·2	22·15
343	1595	8·0, 8·8	22·66	1263	4743	7·6, 8·2	5·43
418	1789	8·5, 9·2	16·10	1281	4782	7·8, 8·9	25·02
434	1827	7·0, 7·8	28·34	1327 A, B	4987	8·0, 9·2	16·13
436	1821	7·0, 8·2	30·21	1327 C	4987	9·0	25·07
447	1869	7·8, 9·0	26·46	1329	4999	8·3, 8·5	27·19
459	1905	7·8, 10·7	12·84	1402	5292	6·8, 8·0	21·09
476	1983	7·5, 8·7	17·58	1409	5336	8·7, 11·2	7·79
497	2053	8·5, 10·7	14·32	1449	5478	8·5, 8·7	35·99
537	2158	8·1, 11·2	14·99	1472	5557	7·8, 8·5	33·74
613 A	2430	7·7, 8·7	19·83	1484	5595	8·7, 12·0	11·95
629	2558	8·2, 11·2	13·16	1486	5597	7·5, 8·8	28·32
634	2548	4·5, 7·9	34·04	1516	5706	7·0, 7·5	9·93
649	2546	7·0, 8·7	21·59	1588	6006	8·5, 8·7	16·49
651	2560	8·0, 10·0	10·81	1594	6012	8·7, 10·5	16·95
653 C	2591	5·0, 7·2	14·65	1602	6035	7·5, 9·0	13·00
704	2738	7·2, 9·5	26·53	1607	6063	7·8, 8·3	33·07
735	2807	8·5, 9·0	30·92	1641	6161	10·0, 10·5	6·14
771	2901	9·0, 9·2	26·34	1644	6175	8·7, 9·2	21·82
782	2923	7·8, 8·3	36·16	1664	6225	7·7, 8·8	17·10
809	3015	7·7, 8·8	25·70	1678	6263	6·3, 7·0	32·60
853	3183	7·8, 8·3	24·06	1682	6284	6·7, 9·0	33·65
859	3190	8·0, 8·5	31·42	1689	6307	6·7, 9·0	28·66
861 A	3194	7·8, 8·2	67·14	1703	6333	8·0, 11·0	22·65
878	3267	7·2, 11·0	16·19	1746	6493	7·7, 10·3	29·62
889	3294	7·2, 9·5	22·04	1801	6716	9·0, 10·5	18·44
909	3385	8·0, 10·9	12·97	1847	6881	8·5, 9·8	18·73
943	3495	8·5, 9·0	15·46	1893	7066	8·4, 10·0	21·60
965 D	3604	8·3, 8·7	47·02	1901	7098	7·7, 9·5	30·34
978	3685	7·0, 9·8	14·78	1983	7422	8·7, 10·8	17·44

TABLE III.—*continued.*

Σ.	β.G.C.	Mags.	Sep.	Σ.	β.G.C.	Mags.	Sep.
			″				″
1993	7466	8·2, 8·2	33·96	2646	10009	7·0, 8·8	24·70
2007	7500	6·5, 8·0	31·97	2649	10005	7·7, 8·8	26·08
2010	7514	5·0, 6·0	31·21	2658 C	10044	7·0, 10·2	32·07
2082	7714	4·0, 10·7	22·39	2679	10182	7·4, 8·7	21·90
2120	7858	6·4, 9·2	3·83	2681 CD	10196	8·0, 11·0	21·97
2145	7935	8·0, 9·5	9·79	2690	10281	7·0, 7·2	14·19
2150	7968	9·3, 10·2	8·08	2703 C	10361	7·6, 7·6	66·72
2192	8107	7·5, 9·9	10·41	2704	10363	3·0, 11·0	32·48
2251	8247	8·2, 11·2	14·41	2708	10402	7·0, 8·7	11·25
2253	8255	7·5, 10·2	18·06	2734	10595	8·2, 8·7	28·50
2268	8325	8·0, 9·0	18·13	2753	10723	7·5, 11·0	31·15
2269	8330	7·5, 10·8	20·10	2754	10725	8·0, 8·7	34·58
2295	8428	8·2, 10·3	11·60	2759	10741	8·5, 9·5	14·57
2306	8502	7·2, 7·9	12·81	2760	10746	7·3, 8·1	13·66
2311	8512	8·9, 9·9	8·65	2777	10829	4·1, 10·2	27·40
2340	8643	8·3, 9·2	21·51	2778	10838	8·4, 10·6	21·19
2342	8654	5·7, 8·5	26·91	2779	10835	8·5, 8·5	19·22
2345	8660	8·4, 10·1	7·38	2803	11032	7·4, 9·0	23·23
2346	8673	7·5, 9·0	15·41	2828 A	11272	8·0, 9·0	23·79
2393	8792	7·3, 10·0	10·42	2860	11471	7·7, 9·3	3·32
2396	8824	7·7, 11·2	11·74	2865	11504	8·5, 9·0	16·36
2400	8830	8·1, 10·6	2·85	2877	11592	6·4, 9·6	7·63
2416	8902	8·0, 10·2	15·61	2895	11657	8·5, 10·0	4·85
2420	8906	4·6, 7·6	30·33	2915	11789	8·5, 8·7	12·27
2421	8925	8·0, 8·7	21·15	2919	11796	9·0, 10·5	14·30
2424	8940	5·7, 9·2	18·66	2941	11952	7·5, 10·2	8·73
2427 A	8943	8·5, 9·0	44·24	2943	11967	6·0, 9·2	30·70
2434 A	8986	7·9, 8·4	25·56	2944 C	11968	7·0, 8·2	55·64
2436	8983	7·4, 8·1	34·58	2954	12044	9·0, 9·0	36·73
2442	9001	8·0, 9·5	23·05	2959	12069	6·5, 10·5	15·66
2444	9003	8·5, 10·2	24·78	2976 C	12172	8·3, 8·8	15·89
2456	9041	8·2, 8·2	29·07	3008	12340	7·0, 8·0	7·54
2472	9075	7·5, 9·2	17·14	3028	12479	7·0, 9·5	19·50
2507	9243	8·2, 9·3	23·86	3039	12552	7·3, 9·7	30·33
2514	9251	9·0, 11·3	7·39	3041 A	12563	7·3, 8·2	71·09
2515	9282	8·0, 9·0	18·74	3073	5878	8·2, 12·0	10·73
2521	9308	5·5, 10·3	22·65	3093	7237	8·0, 9·2	33·38
2612	9814	7·8, 8·8	36·59	3119	4662	8·0, 11·0	24·82
2615	9834	7·2, 10·1	10·82	3127	7922	3·0, 8·1	25·85

Probably Optical Systems.

Σ.	β.G.C.	Mags.	Sep.	Σ.	β.G.C.	Mags.	Sep.
			"				"
596	2351	8·0, 10·2	11·12	1830	6801	8·5, 9·8	4·84
829 A	3096	9·0, 10·7	16·50	2017	7542	7·7, 8·4	25·03
830 A	3113	8·2, 8·7	12·82	2096	7758	6·0, 9·3	22·25
846	3164	8·2, 10·7	12·58	2169	8032	8·0, 10·0	14·85
876	3250	8·5, 11·0	7·72	2184	8065	6·3, 11·2	21·42
1025	3858	7·5, 7·8	22·67	2453	9035	8·2, 10·7	15·13
1046	3913	8·6, 11·7	12·07	2564	9521	8·5, 10·2	10·78
1091	4062	8·2, 8·7	28·59	2838	11327	6·0, 8·8	21·65

On the Absorption of Radiation inside a Star. By A. S. Eddington, M.A., M.Sc., F.R.S., Plumian Professor.

1. In former papers * I have shown that the coefficient of absorption of radiation by stellar material can be determined from the theoretical conditions of equilibrium of a gas-sphere, together with observational knowledge of the luminosities of stars. I have in this way arrived at the numerical value $k = 20$ to 40 C.G.S. units; and this value seems to vary very little from one star to another, at least in the giant stage. The value 40 indicates a remarkably high opacity, so great that a layer of stellar material of atmospheric density 1 cm. thick would absorb $\frac{1}{20}$ of the radiation traversing it; but this is not so surprising when it is realised that the high-temperature radiation inside a star is of short wave-length approximating in character to X-rays. The comparative constancy of k under varying physical conditions is more puzzling since it is not foreshadowed by anything in our laboratory knowledge of the absorption of X-rays.

In the six years since these results were reached I had given much consideration to the possible reasons for the approximate constancy and the particular numerical value of k, but had not been able to get beyond vague possibilities. But I think that the present paper gives a mode of looking at the problem which clears up most of the mystery of this constant. It leads to the conclusion that—

When an electron encounters an ionised atom it will be captured if, and only if, it actually hits the nucleus of the atom.

From this simple, and not unlikely, hypothesis we obtain a value of the absorption-coefficient agreeing closely with the observed value, and we further account for its approximate constancy from star to star. Alternatively we may use the observed value of k to determine the conditions of capture of an electron, and so obtain a fairly secure proof

* *Monthly Notices*, 77, 16, 596; 79, 22. I have collected these results, with some additions, in "Das Strahlungsgleichgewicht der Sterne," *Zeitschrift für Physik*, 7, 351 (1921), which gives the most complete account of the theory.

of the approximate truth of the hypothesis. The numerical agreement encourages us to accept the law of dependence of k on physical conditions which results from the above hypothesis (k is only *approximately* constant) and proceed to a higher approximation in astronomical applications of the theory of radiative equilibrium.

It is possible that various complications may occur (some of which are discussed in this paper) when an electron approaches the nucleus very closely; and the above statement of the conditions of capture may need to be modified in points of detail. But the numerical agreement seems to show that this explanation is on the right track; and a more exact formulation of the condition of capture may be left for a subsequent discussion of both the astronomical and laboratory evidence. The laboratory evidence on the subject relates to β particles, which unfortunately have considerably higher speeds than the electrons in the stars; but it is hoped that it may throw some light on doubtful points.

I shall work out the connection of the absorption-coefficient k with the conditions of electronic capture, first numerically for particular data, and afterwards symbolically in order to study permissible changes in the data. The advantage of the numerical solution is that it indicates the order of magnitude of the quantities concerned, which is a necessary guide to the procedure to be adopted.

2. Owing to the high ionisation within a star, it is fairly certain that the molecular weight m (*i.e.* average weight of the independent particles) is small. It is, I think, certainly less than 6, and it cannot be less than 2, which is the limiting value for complete ionisation. In former papers I have used the value 2·8 (strictly $2\sqrt{2}$), and shall continue to do so here. The astronomically determined value of the coefficient of absorption k is then 23 C.G.S. units. Since k is only approximately constant, it is desirable to consider to what conditions the average value 23 is supposed to correspond. It was based on the datum that a giant star of mass 1·5 × sun has an absolute bolometric magnitude $-0^{m}\cdot3$. We should therefore take a star of mass 1·5. If k is not constant from type to type, its average value must be supposed to refer to the middle of the range, viz. type G. I have given a table of density and temperature in a giant star of mass 1·5 and type G (*Zeitschrift f. Physik.*, 7, 375 *), and I assume that this is the star to which the value $k=23$ most nearly applies. I have also shown that if k is not the same in all parts of the star, the observed k will be an average value taken in a way which gives much greater weight to the highest temperatures. Accordingly I use the second line in the table above mentioned (the first line giving the central temperature 6,590,000°). Thus my fundamental data are

$$\rho = \cdot0678, \qquad T = 5{,}640{,}000^{\circ},$$

together with

$$k = 23, \qquad m = 2\cdot83.$$

We shall see later that the choice of a particular region of a particular star does not make much difference; but when special interest is attached to a numerical agreement it is desirable to avoid temptation to "improve"

* In the corresponding table, *Ap. J.*, **48**, 213, the temperatures are too low in the ratio $1/\sqrt{2}$.

the agreement by arbitrary selection of data, and I have therefore explained fully the principle of my choice. I proceed to calculate some consequences of these data.

The radiant energy passing through a sq. cm. per sec. is $a\mathrm{CT}^4$, where a is the constant of Stefan's law $= 7{\cdot}06 \cdot 10^{-15}$ and C the velocity of light. The amount is

$$2{\cdot}14 \cdot 10^{-23} \text{ ergs.}$$

The energy absorbed per sec. per gm. of material is k times this, viz.

$$4{\cdot}93 \cdot 10^{24} \text{ ergs.}$$

By Planck's formula the average energy of a quantum of radiation at temperature T is $2{\cdot}70$ RT, where R is the physical constant $1{\cdot}346 \cdot 10^{-16}$. Thus the average energy of a quantum is

$$2{\cdot}05 \cdot 10^{-9} \text{ ergs.}$$

Hence, by division, the number of quanta absorbed per gm. per sec. is

$$2{\cdot}40 \cdot 10^{33}.$$

Each time a quantum is absorbed an electron is shot out of the atom and becomes free until it is recaptured. Thus the number of quanta absorbed is equal to the number of separate electron "lives," or to the number of free electrons multiplied by the number of "lives" per second enjoyed by each electron.

The number of hydrogen atoms in a gm. is $6{\cdot}02 \cdot 10^{23}$; hence the number of independent particles of average weight $2{\cdot}83$ in a gm. is $2{\cdot}13 \cdot 10^{23}$. This will be approximately the number of free electrons, since the electrons greatly outnumber the atoms. We may make the slight correction necessary to allow for the fact that a small proportion of the independent particles are not electrons, by knocking off the decimal figures; that allows sixteen electrons to each atom (a probable proportion) and the number of free electrons is then

$$2{\cdot}0 \cdot 10^{23} \text{ per gm.}$$

Accordingly, dividing the number of quanta by the number of electrons, each free electron has an average number of lives,

$$1{\cdot}20 \cdot 10^{10} \text{ per sec.}$$

The average speed of a hydrogen molecule at 273° C. absolute is $1{\cdot}694 \cdot 10^5$ cm. per sec. To obtain the average speed of an electron we must multiply by $\sqrt{3660}$ for the difference of mass and by $\sqrt{(\mathrm{T}/273)}$ for the difference of temperature. The average speed of the electrons is thus

$$\mathrm{V} = 1{\cdot}47 \cdot 10^9 \text{ cm. per sec.}$$

Dividing the distance travelled per sec. by the number of lives per sec., we obtain the mean path during one life,

$$\lambda = 0{\cdot}122 \text{ cm.}$$

Apparently we make no allowance for the time wasted in a captured state by the electron. This is justified because, whilst an electron is captive, it is replaced by another electron enjoying freedom. If, of the electrons concerned in the ionisation, p are describing free paths and q are captive at any one moment, the number of electrons to be considered is $(p+q)/p$ times the number of free electrons at any moment, and the free life must be proportionately lengthened. This factor cancels out the factor $p/(p+q)$ which would allow for the proportion of time spent in captivity.

The "free path" here found is the length of path described until an encounter with an atom occurs of such a character as to lead to capture. There may in the meantime have been a large number of encounters not resulting in capture. We may conveniently introduce the idea of a *target.* If on the average only one in a million of the encounters with atoms leads to capture, we take the area of the target to be one-millionth of the area of the atom. We may either regard the target as a definite region of the atom, which the electron has to hit in order to be captured, or we may regard it as a statistical equivalent for the atom describing its average capturing power. Our data permit us to calculate the radius of the target.

In the theory of gases the mean free path is $1/\pi\nu\sigma^2\sqrt{2}$, where σ is the diameter of the molecule and ν the number of molecules per c.c. This refers to encounters of equal spheres. Here we have an electron (practically a point) encountering a much larger target; and, moreover, the targets, being heavy atoms, have velocities negligible compared with the velocities of the electrons. The free path in this case is

$$\lambda = 1/\pi\nu\sigma^2 \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where σ is now the *radius* of the target. To find ν (the number of atoms per c.c.) we must know the atomic weight. We shall adopt 56 (iron).* Using the given density $\rho = \cdot 0678$, we easily find

$$\nu = 7\cdot 28 \,.\, 10^{20},$$

which, with the value of λ already found, gives by (1)

$$\sigma = 5\cdot 98 \,.\, 10^{-11} \text{ cm.}$$

The target is very much smaller than the atom, which has a radius of the order 10^{-8} cm. Only a small proportion of encounters with atoms result in capture. We shall assume that the target is an actual sphere at the centre of the atom, and not merely a statistical conception. It seems unlikely that the satellite electrons could play any part in capture since they repel the electron from themselves. Owing to the positive charge of the nucleus, electrons hitting the target will be drawn in towards it; and an electron which would if undisturbed have passed at a distance σ from the centre will actually approach to a smaller distance d. We shall call d the radius of the *true target,* so that all

* By varying the element chosen, considerable difference can be made in the next few numerical results, but we shall find that the final result (d) is practically independent of the element chosen.

electrons which were aimed at the apparent target will actually hit the smaller true target. We have now to calculate d.

Applying the usual astronomical equation

$$h^2 = \mu l = \mu a(e^2 - 1) \quad . \quad . \quad . \quad . \quad (2)$$

we have for an electron of initial velocity V and undisturbed distance σ from the centre

$$h = V\sigma \quad . \quad . \quad . \quad . \quad . \quad (3)$$

also the distance of nearest approach is

$$d = a(e-1) \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Also σ, being the perpendicular on the asymptote, is equal to the minor axis of the hyperbola, so that

$$\left.\begin{aligned} \sigma^2 &= a^2(e^2-1) \\ \text{and} \qquad \frac{d^2}{\sigma^2} &= \frac{e-1}{e+1} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (5)$$

If ϵ is the electronic charge and $N\epsilon$ the nuclear charge, so that N is the atomic number, m the mass of the electron,

$$\mu = N\epsilon^2 C^2/m \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Hence by (2), (3), and (4)

$$d = \frac{\sigma^2}{N(1+e)} \frac{V^2}{C^2} \frac{m}{\epsilon^2} \quad . \quad . \quad . \quad . \quad (7)$$

Now $\frac{2}{3}\epsilon^2/m$ is the quantity usually taken as the radius of the electron, viz. $1{\cdot}87 \cdot 10^{-13}$ cm. For iron $N = 26$. We can put $1+e=2$ with sufficient approximation (as will be verified later). The calculation of (7) then gives

$$d = 5{\cdot}9 \cdot 10^{-13} \text{ cm.}$$

Hence $d/\sigma = {\cdot}01$, so that by (5) $e = 1{\cdot}0002$, showing that the approximation $1+e=2$ is adequate.

The capture of the electron is therefore determined by its centre hitting some part of a sphere of radius $5{\cdot}9 \cdot 10^{-13}$ cm. The nucleus of an iron atom consists of 56 protons and 30 electrons packed tightly together. I think it is generally believed that the packing is so close as to squeeze the electrons out of shape; at least this seems to be the case in the helium atom. The protons occupy practically no room, and we could pack 27 electrons into a cube of half-edge $5{\cdot}7 \cdot 10^{-13}$ cm., so that this might be taken as about the average radius of an atomic nucleus. The radius of the true target is thus as nearly as possible equal to the radius of the nucleus; and the conclusion follows that the nucleus constitutes the target. Presumably the impact is inelastic, and the electron having discharged its surplus energy drops back tamely into a vacant place in one of the satellite rings. It may be added that if Newtonian ideas are applicable an electron would certainly lose energy

by encountering the system of 86 attracting and repelling particles which constitutes the iron nucleus.

If our figures could be taken as exact, we might decide that a graze would not suffice for capture and that the impact must be at least a half-ball shot (which seems reasonable). But I cannot stress the agreement to this extent; and when certain corrections have been applied and uncertainties considered, we may be well satisfied if the target and nucleus agree to within a factor 3 or 4.

One of these corrections may be considered at once, since there is no uncertainty in calculating it. The electrons do not all move with the average speed. If the true target is fixed, the area $\pi\sigma^2$ of the apparent target varies as V^{-2} by (7). The number of captures per second accordingly varies as V^{-1}. In taking an average value of V we must represent correctly the total number of captures, since this is determined from the observed value of k; accordingly it is V^{-1} which must be averaged, *i.e.* the harmonic mean speed, not the arithmetic mean speed, is to be taken. For Maxwell's law the harmonic mean is $\pi/4$ times the arithmetic mean; hence d must be reduced in the ratio $\pi/4$. Accordingly $d = 4{\cdot}6 \cdot 10^{-13}$ cm.

3. By following the same procedure as in the foregoing numerical calculation, we can obtain the symbolic expression for the radius d of the true target.

Let A be the atomic weight of the material (in terms of hydrogen).

N the atomic number.

m the molecular weight (*i.e.* average weight of atoms and free electrons together).

H the mass of a hydrogen atom $= 1{\cdot}662 \cdot 10^{-24}$.

u the *harmonic* mean velocity of an electron at 1° absolute $= 4{\cdot}87 \cdot 10^{5}$.

$b = \frac{3}{2} \times$ radius of an electron $= 2{\cdot}81 \cdot 10^{-13}$.

The physical constants $a = 7{\cdot}06 \cdot 10^{-15}$, $R = 1{\cdot}346 \cdot 10^{-16}$, $C = 3 \cdot 10^{10}$ are defined as before.

The total number of quanta absorbed per gram per second is

$$\frac{aCkT^4}{2{\cdot}7RT}.$$

The number of free electrons is $1/Hm$ (subject to a small correction of about 6 per cent. to allow for the number of atoms). Hence the number of "lives" of an electron per second is

$$\frac{HmaCkT^3}{2{\cdot}7R}.$$

From this, since $V = uT^{\frac{1}{2}}$,

$$\pi\nu\sigma^2 = \frac{1}{\lambda} = \frac{HmaCkT^{\frac{5}{2}}}{2{\cdot}7Ru}.$$

Also $\nu = \rho/HA$, so that

$$\sigma^2 = \frac{H^2AmaCkT^{\frac{5}{2}}}{2{\cdot}7\pi Ru\rho} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Then by (7)

$$d = \frac{H^2 au}{2{\cdot}7\pi RCb} \cdot \frac{A}{2N} \cdot mk\frac{T}{\rho} \quad . \quad . \quad . \quad . \quad (9)$$

$$= [\bar{4}0{\cdot}994]\frac{A}{2N}mk\frac{T^{\frac{1}{2}}}{\rho}.$$

The bracketed number is the logarithm of the coefficient.

Apart from the above-mentioned increase of about 6 per cent., this expression is rigorous until we reach extremely high electronic velocities, for which the factor 2 in the denominator would have to be replaced by $(1+e)$. But this would only happen at high temperatures much beyond those occurring in the stars.

From this formula we can see how the previous value of d will be modified by any change in the assumed composition and physical state of the star.

4. The value of d is practically independent of any assumption as to the chemical constitution of the star. The properties of the atom are involved only in the factor A/2N, which is nearly equal to 1 for every element except hydrogen. As an example of an element of high atomic weight we may take gold (A = 197, N = 79), for which this factor is 1·25 against 1·08 for iron. If, however, we substitute gold for iron, we should for consistency increase the adopted radius of the nucleus in the ratio $\sqrt[3]{(118/30)}$ or 1·58. The agreement of the target with the nucleus is not much disturbed even by this extreme change of chemical composition.

For the 91 elements (excluding hydrogen) A/2N ranges between 1·00 and 1·29. If we adopt A/2N = 1·14, the extreme possible error in d is about 12 per cent.

The value of d is more sensitive to possible changes in the assumed degree of ionisation, which is indicated by m. A small change in m is somewhat magnified by consequential changes in k and T, the values employed above having been based on the adopted value 2·83 for m. In my later work I have been led to prefer a somewhat higher value of m, finding that it gave better accordance with astronomical conditions. The most precise test is based on the known distribution of the masses of the stars. It seems clear that the reason why masses several times larger than the sun's mass are rare, is that the large radiation-pressure endangers their stability; and most masses will range about the point where radiation-pressure ceases to be a serious danger. For $m = 4$, radiation-pressure amounts to 0·23 of gravitation for mass 1, and 0·11 of gravitation for mass $\frac{1}{2}$; whereas for $m = 2{\cdot}83$ the corresponding figures are 0·11 and 0·036. In the latter case it would seem that a mass equal to the sun's ought to be very nearly safe; and it is difficult to account for so many stars with mass less than 1 and not infrequently less than $\frac{1}{2}$. For $m = 4$ the radiation-pressure in this range is still reasonably important; perhaps a slightly larger value of m would account for the facts even better. The weakness of this argument is that the break-up of stellar material must in general have occurred before the condensation had proceeded very far, so that we are deriving by this test a value of m at an early period in the star's history which may have

altered as the temperature rose. But there are in addition certain other astronomical indications (the maximum observed luminosities of stars) which are best satisfied by a value of m near to, if not greater than, 4.

Eggert* has concluded from thermodynamical considerations that iron under stellar conditions would at most lose its two outer rings of electrons (16 electrons). This gives $m = 3{\cdot}3$ (or perhaps greater). The result, however, depends on the chemical element assumed.

It seems probable that m should be increased a little, and I adopt $m = 4{\cdot}0$ as the most likely value on the present evidence. The temperatures must then be increased in the ratio 1·16, and k in the ratio 1·84. The whole effect is that d is increased in the ratio 4·43.†

The value of k which we have used corresponds to the assumption that the heat liberated (by sub-atomic energy or by contraction) is generated uniformly per unit mass; if not, a small correction is required. Using the notation of my previous papers, k must be replaced by ηk. The value of η near the centre of the star (which is the region here considered) can be calculated if the law of generation of heat is known. On the contraction hypothesis, or if sub-atomic energy is liberated proportionately to the temperature, we have $\eta = 1{\cdot}7$ (the ratio of the central temperature to the mean temperature). Probably the proportion of heat liberated at higher temperatures is greater than that given by this law, so that η is larger, but it would require very extreme conditions to raise it at all considerably. We may perhaps adopt $\eta = 2{\cdot}5$ as the best estimate that can be made.

It is possible that the factor 2·7 in (9), depending on the mean energy of a quantum, may also have to be altered slightly. If the absorption is not uniform for different wave-lengths, the mean quantum absorbed will not be the same as the mean quantum of the free radiation. ‡

Accordingly I apply the following four corrections to the original value of d, $5{\cdot}9 \cdot 10^{-13}$:—

1. Multiply by $\pi/4$ to reduce to harmonic mean velocity.
2. Multiply by 1·14/1·08 to reduce to a mean element instead of iron.
3. Multiply by 4·43 to reduce to ionisation $m = 4$.
4. Divide by 2·5 for the concentration of liberation of energy towards the centre.

The net result is to multiply d by 1·47, giving

$$d = 8{\cdot}7 \cdot 10^{-13} \text{ cm.}$$

* *Phys. Zeits.*, **20**, 570.

† The increase of d is in the ratio $m(1-\beta)^{\frac{1}{3}}$, where $(1-\beta)$ is the quantity defined in my former papers. For $m = 4{\cdot}9$, the increase is in the ratio 8·6; for $m = 5{\cdot}9$, increase 14·3; for $m = 6{\cdot}9$, increase 22·0 (compared with $m = 2{\cdot}8$ in all cases). It must be remembered that these small changes of m correspond to enormous changes in the assumed degree of ionisation; for example, iron which has lost only the outer ring of eight electrons corresponds to $m = 6{\cdot}2$, as against $m = 3{\cdot}3$ for the loss of two rings.

‡ The original mean energy of a captured electron (allowing for the fact that the slow-moving electrons are the more likely to be captured) is RT. After capture it has a negative energy of roughly the same amount, but it is not calculable accurately. The difference is the average emission per capture.

The only considerable uncertainty arises from the doubtful value of m, and from uncertainties of the observational data used in determining k; but it is scarcely likely that the factor of uncertainty can exceed 3.

5. We shall now regard the target radius d as a fixed datum, presumably representing the radius of the atomic nucleus, and calculate the dependence of k on the physical conditions of the material. Then by (9) we see that

$$k \propto \rho T^{-\frac{7}{2}}.$$

I have shown that throughout any giant star, and for all giant stars of the same mass but arbitrary densities, $\rho \propto T^3$. Accordingly

$$k \propto T^{-\frac{1}{2}}$$

Consider first the variation of k in different parts of the same star. Excluding the outermost 5 per cent. of the mass, the temperature ranges from $1{\cdot}4 \cdot 10^6$ to $6{\cdot}6 \cdot 10^6$ degrees. Thus k decreases from near the edge to the centre in the ratio 2·2 : 1. The assumption of approximate constancy of k in the theory of radiative equilibrium is thus justified. As a matter of fact even this small change of k has been discounted in the calculations of the theory. The assumption in the calulations is that, not k, but ηk is constant. Whilst k is decreasing from 2·2 to 1, η is increasing from 1 to at least 1·7 and probably rather higher. Although the law of change is not quite the same for the two factors, the assumed constancy of ηk will be a remarkably close approximation.

Next considering the variation of k in stars of the same mass but in different stages of evolution, we have $k \propto T_c^{-\frac{1}{2}}$, where T_c is the central temperature. According to the theory of radiative equilibrium, if T_0 is the effective temperature, $T_0^4 \propto T_c^2/k \propto T_c^{\frac{5}{2}}$, so that $k \propto T_0^{-\frac{1}{5}}$. The total radiation L of a giant star of given mass is inversely proportional to k. Hence

$$L \propto T_0^{\frac{1}{5}},$$

and the absolute magnitude (bolometric) becomes

$$M = \text{const.} - 2 \log_{10} T_0.$$

Thus, for example, between type M(3000°) and type A(12000°) the star becomes 1·2 bolometric magnitudes brighter.

Statistically there is some evidence of a general increase of brightness of the hotter giant stars; but it is scarcely possible to disentangle the effect here predicted from changes due to their greater average mass, and from a decrease of brightness due to rather high density involving departure from the ideal gas-laws. In former papers I have concluded that the observational evidence showed that the variation of k (if any) is not more rapid than $T^{-\frac{1}{2}}$ or $T^{\frac{1}{2}}$. The theoretical law here found coincides with one of these limits.

In comparing giant stars of different mass, and in comparing giant stars with dwarf stars, ρ is no longer proportional to T^3. The law $k \propto \rho T^{-\frac{7}{2}}$ will introduce more considerable changes in the part of the

theory relating to these subjects; but this is too wide a field to enter on in this present paper.

6. I have calculated the distance d assuming that the nucleus attracts the electron according to the inverse-square law even at the small distances considered. I should scarcely have dared to treat the atomic model so realistically were it not for the strong evidence afforded by Rutherford's experiments on the scattering of α particles. He has shown that the inverse-square law is valid to within an extremely small distance from the nucleus. Even more closely germane to the present problem is the work of Crowther and Schonland* on the scattering of β particles. Their work is actually a study of the paths of electrons passing close to an atomic nucleus, only their electrons move with a speed considerably greater than ours. They find more serious deviations from the inverse-square law; but, so far as I can make out, the deviation is most pronounced for the highest speeds and tends to disappear when the initial speed becomes small compared with that of light, as is the case for the electrons which we have to consider.

The velocity V of the free electrons is about $\frac{1}{20}$ of the velocity of light, so that the "relativity-corrections" are inappreciable; but as the electron draws close to the nucleus its velocity and acceleration are enormously increased, so that it becomes a question whether we ought not to take account of two phenomena which may perhaps occur, (1) the increase of mass with velocity, (2) the radiation of energy from an accelerated electron. It happens that either of these phenomena can be taken into account quite rigorously if desired; the only difficulty is that it is by no means certain that they are to be expected theoretically.

(1) *Change of Mass with Velocity.*—The mass of a system is simply another aspect of its energy, and the increase of mass with velocity is due to an increase of energy. But in our problem the energy of the electron (or at any rate the energy of the whole system) does not change; there is gain of kinetic (magnetic) energy but loss of potential (electrostatic) energy. The mass of the atom + electron must remain constant; but it is, of course, possible that this mass may be redistributed. The difficulty is to know how the potential energy ought to be divided between the two bodies. It is scarcely possible to decide this question by present theories. We have hitherto worked out the results on the view that the energy and mass associated with the electron remain constant; we shall now work them out on the view that the electron gains mass by its kinetic energy, and that the loss of potential energy is in the part of the field belonging to the heavier nucleus.

Admitting change of mass with velocity, I have shown † that the equation of the orbit is

$$\frac{d^2u}{d\theta^2}+u=\frac{\mu}{h^2}\frac{\mathrm{m}}{\mathrm{m}_0} \quad . \quad . \quad . \quad . \quad (10)$$

* *Proc. Roy. Soc.*, 100A, p. 526 (1922).

† *Phil. Mag.*, 34, p. 322 (1917), equation (3). The unexpected inversion of the factor m/m_0 should be noticed. The increase of mass with velocity *increases* the curvature of the path. A similar problem has been treated by C. G. Darwin, *Phil. Mag.*, 25, 201 (1913), and it can be shown that his formulæ and mine are equivalent.

where m is the actual mass and m_0 the mass at rest. Here $u = 1/r$, and the force is $\mu m_0/r^2 (\mu = N\epsilon^2 C^2/m_0)$. The original velocity V is small compared with the velocity of light, so that m_0 may be taken as the mass when $r = \infty$. At a finite distance r the potential energy decreases by $\mu m_0/r$, so that this is the increase of kinetic energy, and the increase of mass is $\mu m_0/rC^2$. Hence

$$m = m_0 + \frac{\mu m_0}{rC^2}.$$

Accordingly (10) becomes

$$\frac{d^2u}{d\theta^2} + \left(1 - \frac{\mu^2}{h^2C^2}\right)u = \frac{\mu}{h^2} \quad . \quad . \quad . \quad . \quad (11)$$

Let

$$p^2 = 1 - \frac{\mu^2}{h^2C^2},$$

then the solution is

$$u = \frac{\mu}{p^2h^2} + A\cos p\theta.$$

Or, if

$$l = p^2h^2/\mu \quad . \quad . \quad . \quad . \quad . \quad (12)$$

$$\frac{l}{r} = 1 + e\cos p\theta \quad . \quad . \quad . \quad . \quad (13)$$

e being an arbitrary constant. For $p = 1$, this gives the usual hyperbola; as p decreases we have curves of more or less hyperbolic shape with asymptotes given by $\cos p\theta = -1/e$. The perpendicular from the centre of force on an asymptote is found to be

$$\sigma = l/p\sqrt{(e^2 - 1)} \quad . \quad . \quad . \quad . \quad (14)$$

The singular orbit when $p = 0$ is of special interest. It is a spiral

$$u = \frac{\mu}{2h^2}\theta^2.$$

A particle following this path reaches the centre of force by a spiral track. We may take this value $p = 0$ as dividing the uncaptured from the captured particles. (This would be strictly correct for an infinitely small nucleus, but owing to its finite size, particles for which p is a little greater than 0, will also hit the nucleus; this correction for finite size of nucleus is included in equation (16) below.)

Since $h = \sigma V$, the condition $p = 0$ becomes

$$\sigma = \frac{\mu}{CV} = Nb\frac{C}{V},$$

which by (8) leads to the condition,

$$\tfrac{1}{2}Nb = \frac{H^2au}{2\cdot 7\pi RCb} \cdot \frac{A}{2N} \cdot mk\frac{T^{\frac{1}{2}}}{\rho} \quad . \quad . \quad . \quad (15)$$

This is the same as (9) except that $\frac{1}{2}Nb$ has replaced d on the left side of the equation. Our value of d, arrived at after applying all corrections, was approximately equal to $3b$; hence the effect of allowing a change

of mass with velocity is to increase the predicted value of k in the ratio N/6. For iron this factor is about 4, which is perhaps not outside the limits of uncertainty of the data.

A more exact formula which allows for the finite radius d of the nucleus is easily obtained. The distance of closest approach is by (13) and (12),

$$d = \frac{l}{1+e} = \frac{p^2 h^2}{\mu(1+e)},$$

which, after a little reduction, leads to

$$d + \tfrac{1}{2}Nb = \frac{H^2 au}{2\cdot 7\pi RCb} \cdot \frac{A}{2N} \cdot mk\frac{T^{\frac{1}{2}}}{\rho} \quad . \quad . \quad . \quad (16)$$

where we have put $(1+e)=2$ as before, this being still a very close approximation.

The important point is that the law of dependence of k on the density, temperature, and molecular weight, is unaltered. We merely increase the opacity by a constant factor, which cannot be exactly predicted without knowledge of the chemical constitution of the star, but is fairly certain not to exceed 5.

(2) *Radiation from an accelerated Electron.*—According to Larmor's investigation the energy radiated by an electron having acceleration Γ is

$$\frac{2}{3}\frac{\epsilon^2}{C}\int \Gamma^2 dt \quad . \quad . \quad . \quad . \quad . \quad (17)$$

or since $\Gamma = \mu/r^2$, and $dt = r^2 d\theta/h$, the whole loss of energy during the approach and recession is

$$\frac{2}{3}\frac{\epsilon^2\mu^2}{Ch}\int_{-\pi}^{\pi}\frac{d\theta}{r^2} = \frac{2}{3}\frac{\epsilon^2\mu^2}{Chl^2}\int_{-\pi}^{\pi}(1+e\cos\theta)^2 d\theta,$$

or, since e is approximately unity,

$$= \frac{2\pi\epsilon^2\mu^2}{Chl^2} \quad . \quad . \quad . \quad . \quad . \quad (18)$$

If this loss exceeds the energy $\frac{1}{2}mV^2$ originally possessed by the electron before the encounter, the electron will not be able to retreat from the system and must remain captive. There is no longer any need actually to hit the nucleus; the capture is effected without violent disturbance through continuous radiation of energy. The limiting condition of capture is thus

$$\tfrac{1}{2}mV^2 = \frac{2\pi\epsilon^2\mu^2}{Chl^2} = \frac{2\pi\epsilon^2\mu^4}{Ch^5} \quad . \quad . \quad . \quad . \quad (19)$$

since $h^2 = \mu l$. Since $h = \sigma V$, this becomes

$$V^7 = \frac{4\pi\mu^5}{NC^3\sigma^5} = \frac{4\pi N^4 b^5 C^7}{\sigma^5}.$$

Hence

$$\frac{\sigma}{b} = (4\pi)^{\frac{1}{7}} N^{\frac{4}{7}} (C/V)^{\frac{2}{7}} \quad . \quad . \quad . \quad . \quad (20)$$

Then by (8) we obtain the formula

$$mk\frac{T^{3\cdot9}}{\rho}=\frac{2\cdot7\pi(4\pi)^{\frac{2}{3}}R\delta^2}{H^2a}\left(\frac{C}{u}\right)^{\frac{2}{3}}\cdot\frac{N^{\frac{4}{3}}}{A} \quad . \quad . \quad . \quad (21)$$

Using $C/V = 20\cdot4$, $N = 26$, as in our previous work, we obtain from (20)

$$\sigma = 4\cdot3\,.\,10^{-10}$$

which is to be compared with the value obtained in § 2, $\sigma = 5\cdot98\,.\,10^{-11}$. This gives a target area 54 times greater than that previously adopted, so that the coefficient of absorption would have to be correspondingly greater. The discrepancy is reduced a little if we make the necessary changes to molecular weight 4; but it seems clear that the absorption is far too high to be admissible.

For giant stars of the same mass in which $T^3 \propto \rho$, we shall have by (21) $k \propto T^{-\frac{2}{10}}$; and since the effective temperature $T_0 \propto k^{-\frac{1}{4}}T$, we obtain the law $k \propto T_0^{-\frac{2}{15}}$. Accordingly the total radiation L of a star in different stages of evolution varies according to the remarkable law

$$L \propto T_0^{\frac{23}{15}} \quad . \quad . \quad . \quad . \quad . \quad (22)$$

The law might perhaps be accepted by straining the observational evidence a little, if there were no other objection.

If we accept the astronomical evidence that the value of k from equation (21) is too high to be admissible, we have practically a direct proof that an electron on entering an atom cannot radiate energy according to Larmor's formula. For it is clear that the electrons which radiate their whole original energy cannot retain their freedom, and therefore we can assign at any rate a minimum rate of capture of free electrons. The material must absorb the same minimum number of quanta of radiation in order to set free the electrons required to take their place, and therefore a minimum limit to the coefficient of absorption can be definitely given.

The theory of radiation from an accelerated electron has often been criticised. It is well known that it cannot apply to electrons describing Bohr orbits, and we have here rather strong evidence that it does not apply to electrons describing free paths near a nucleus. On the other hand it has been confirmed experimentally for the case of scattering of X-rays by electrons.

According to (7) d varies as σ^2, so that the fifty-fold increase in the apparent target-area means a fifty-fold increase in the pericentric distance. There is no longer a close approach to the nucleus by the captured electrons. It is thus unnecessary to consider the variation of mass with velocity and the radiation of the accelerated electron *simultaneously*; if one of them is significant, the other is negligible.

7. We have now discussed three possible theories of capture of electrons—

(*a*) The electron moving under Newtonian laws hits an atomic nucleus of finite size, and loses its energy by inelastic impact.

(*b*) The electron's mass varies with its velocity according to the usual formula. The capture again occurs by hitting the nucleus, but the size of the nucleus now makes comparatively little difference to the result.

(*c*) The electron is captured by radiating its energy in consequence of its acceleration, according to Larmor's formula. It does not necessarily pass very near the nucleus.

In all three cases the calculated value of the coefficient of absorption is not so very far * from the observed value. The agreement appears to be best for (*a*); (*b*) gives a value about three or four times that which seems to us most likely, but this is perhaps within the limits of uncertainty; (*c*) gives a value which is almost certainly too high to be possible.

The result for (*a*) is practically independent of the chemical constitution of the star except through the radius of the nucleus. For the three theories we have

$$k \propto N^{\frac{1}{3}}, \quad N, \quad N^{\frac{2}{3}}, \text{ respectively,}$$

so that in the last two cases the agreement would be somewhat improved if the star were composed of light elements. But it must be remembered that for light elements the ionisation is likely to be higher than we have assumed, and this tends to neutralise the improvement.

The three theories give the following laws of dependence of k on temperature and density—

$$(a) \text{ and } (b),\ kT^{3\cdot5}/\rho = \text{const.};\ (c),\ kT^{3\cdot9}/\rho = \text{const.}$$

Until recently my attempts to arrive at a theoretical determination of k have been concerned with the mechanism of *absorption*, *i.e.* the disruption of electrons from atoms. They were unsuccessful. Here we appear to have made greater progress by considering the mechanism of *emission*, *i.e.* the capture of electrons by atoms. The manner in which the rate of capture adjusts itself to the rate of disruption is regulated by the number of free electrons. If the rate of capture fell short of the required value, more free electrons would accumulate and this would immediately raise the rate of capture. Thus the applicability of the method depends on the possibility of our being able to estimate for what number of free electrons equilibrium is established. This number is found either (1) by general thermodynamical principles which arrive at the result without knowledge of the details of emission or absorption, or (2) by observational evidence of the effect of these electrons on the equilibrium of a star through lowering the molecular weight.

Summary.

1. From the known value of the stellar coefficient of absorption k I calculate the number of "lives" per second of the free electrons in the star. I hence deduce the area of the atomic target which the electron

* At various stages of the calculation of k, the characteristic of the logarithm often amounts to as much as ± 60. It is with this in mind that I regard the result as "not so very far out" when the final characteristic comes down to within one of the required value.

has to hit in order to be captured by an atom. The target turns out to be small compared with the size of the atom, and presumably represents a small region round the nucleus.

2. Allowing for the drawing in of the electrons under the powerful attraction of the nucleus, I calculate the radius of the "true" target, which is considerably smaller than the apparent target. After applying all corrections, the best value of this radius appears to be $d = 8{\cdot}7 \cdot 10^{-13}$ cm.

3. This value of d is almost independent of the chemical constitution of the star, excluding hydrogen. For all the remaining 91 elements the value will not vary more than 12 per cent. from the mean.

4. The observed average value of k must be taken to correspond to a region near the centre of the star; but actually it makes no serious difference in the calculation of d what part of the star is considered (except the extreme edge), the variations being limited to about 20 per cent.

5. The chief source of uncertainty in d is the degree of ionisation. From this cause a factor of uncertainty 4 or $\frac{1}{4}$ might be required, but I think not a larger factor. There is also some uncertainty of the observational data on which k is based.

6. The radius of the nucleus of an atom of average atomic weight may be taken to be about $6 \cdot 10^{-13}$ cm., agreeing very nearly with the above value of d. It therefore seems probable that the condition of capture is that the electron should actually hit the nucleus. It is presumed that the impact is inelastic, so that the electron loses sufficient energy to prevent it escaping again from the system.

7. Assuming on these grounds that the true target is of a definite size independent of physical conditions, we can reverse the procedure and calculate the dependence of k on physical conditions. It is found that $k \propto \rho/T^{\frac{7}{2}}$. Within a single giant star, and for giant stars of the same mass but in different stages of evolution, $\rho \propto T^3$, so that $k \propto T^{-\frac{1}{2}}$.

8. The approximation $\eta k = \text{const.}$ used in the theory of radiative equilibrium is justified to a very high degree of accuracy, since the small changes of η and k within the star practically counterbalance one another.

9. The change of k from star to star according to the law $T^{-\frac{1}{2}}$ (which is equivalent to $T_0^{-\frac{1}{2}}$) is within the range of possible variation of k, which I have previously laid down as probably consistent with the observational evidence.

10. The effect of change of mass with velocity has been considered. It is uncertain whether it would actually occur under the conditions here discussed. If it does, it will increase the predicted value of k, three or four-fold, but will not change the law $k \propto \rho T^{-\frac{7}{2}}$.

11. The possible radiation of an accelerated electron has also been considered. It leads to results which appear to be inadmissible astronomically.

12. The law of variation of k here found will modify certain applications of the general theory of radiative equilibrium of the stars. These are reserved for subsequent discussion.

Spectroscopic Parallaxes of the Hotter Stars. (*Preliminary Paper.*)

By D. L. Edwards, A.R.C.S., D.I.C.

Introduction.

The spectroscopic method of obtaining stellar parallaxes has developed considerably in accuracy and importance since its origination by Professor Adams. Several observatories are now using the method, and it has become a very important means of obtaining, with comparative ease and rapidity, measures of stellar luminosities and parallaxes which, in the case of more distant stars, may exceed in accuracy the results of the older trigonometrical methods. Up to the present, however, the method has only been available for stars of later types (F_0 to M, with a few A stars), although a method of obtaining the distances of B stars would be of considerable importance owing to the unreliability of trigonometrical measures of such distant objects, and the comparatively small amount of data which we possess concerning them at present. An attempt has therefore been made to extend the method so as to make it applicable to the higher-type stars (Oe_5 to F_0), or to deduce a similar method based on the same principles which could be used for stars of these types.

It seemed evident at first glance that these stars would have to be divided into at least two groups, each of which would require special treatment. In the A stars (among which one might include B_9) only a few important lines are found, apart from hydrogen, and these are mostly spark lines of metals. The B stars, on the other hand, possess strong lines of helium, together with fainter lines of nitrogen, carbon, oxygen, etc., none of which is prominent in A stars. There is therefore very little in common between these two groups except, perhaps, the Mg^+ line at $\lambda 4481$. Later investigation has shown that the Oe stars, although apparently similar to the early B stars, do not conform to the results found for the latter. There are thus three main groups of stars, each of which has been investigated as described in the following paragraph, and the present paper gives the results so far obtained in the case of the B stars.

The Preliminary Investigation.

The negatives used in this investigation were taken with the "McClean" 12-inch prismatic camera of the Norman Lockyer Observatory. The greater number of them were taken by Major W. J. S. Lockyer and the writer, and these were supplemented by some southern stars photographed at the Cape by Dr. F. McClean, F.R.S., with the same instrument. These were presented to the Observatory by his son, Lieut.-Col. F. K. McClean. The prism has an angle of 20°, and the dispersion produced on the plate is such that the distance between H_β and H_δ is 21·4 mm.

In order to discover which lines, if any, were subject to variation with change of absolute magnitude, and which would therefore be of value in parallax determinations, the following procedure was adopted. About 200 negatives of stellar spectra were chosen out of the best of

those available for the particular types under investigation. In each of these negatives the intensities of all the prominent lines (including hydrogen) were measured, together with any fainter lines which might be suspected of variation. The negatives were measured indiscriminately, without any knowledge of their type or absolute magnitude, in order to avoid any possibility of bias; and when possible the measures were checked by using more than one negative of a star.

The next step was to reduce these intensities to some common standard, so that a line in one star could be compared with the same line in any other star, irrespective of any variations in the qualities of the negatives. In all previous work this had been attained by taking a comparison line for each principal line, and regarding the differences of intensity between the lines in each pair thus formed as the quantities to be related to absolute magnitude. In the present case, however, it was considered sufficient (for a first approximation) to take the mean intensity of the four hydrogen lines H_β to H_ϵ as a common standard of comparison for all lines. This mean hydrogen intensity was found for each negative, and the intensities of each of the remaining lines subtracted from it. These differences were then regarded as reduced line intensities which could be compared directly in different stars.

The various lines were then considered individually, and their reduced intensities plotted against (*a*) the spectral types of the stars, and (*b*) their absolute magnitudes (when known). In this way any relation between line intensity and type or absolute magnitude at once became apparent. In the case of one or two lines such a relation was definitely indicated, and in such cases the lines were carefully remeasured and compared with some comparison line chosen as near as possible in the spectrum; the procedure then being similar to that adopted for the cooler types. More accurate curves were thus obtained, from which spectral types or absolute magnitudes could be directly derived.

Method and Accuracy of the Measurements.

The line intensities required for this work were all measured by the "wedge" method, recently described by Major Lockyer.* Only one wedge was employed throughout the series, and the illumination was kept as constant as possible. Five settings were made on each line, and these individual measures were found to differ from the resulting mean by not more than 0·5 wedge divisions. The mean value (described to the nearest tenth of a division) was taken as the required line intensity, and was checked whenever possible by measuring at least two different negatives of the same star.

There were various possible sources of error to be considered, and an attempt was made to eliminate these, or to reduce them to such dimensions that they might safely be neglected. For example, the illumination might be regarded as constant during the measurement of any one negative. Any variations which might occur from day to day would affect all lines equally, so that differences of intensity would remain unaltered, and the effect on the final results would be negligible. This

* *Monthly Notices, R.A.S.*, 82, 226.

was tested by measuring a pair of lines under different conditions of illumination, when the intensity difference of the pair was found to be constant. The question of errors of the wedge, personal equation, and accidental errors in measuring have already been dealt with by Lockyer in the paper mentioned above. For the purpose of this preliminary work they may be disregarded if due precautions and a sufficient number of individual measures are taken. The most likely source of error lies in the variations in quality of the negatives themselves. These are due, in the first place, to variable factors such as exposure (including, indirectly, atmospheric conditions and altitude of the star), the subsequent treatment in the developing and fixing baths, and the presence of moonlight or other diffused light which may give rise to general fog. It was found that for negatives of fairly normal density the results were quite consistent, although slight variations undoubtedly occurred. It was found, for example, that if two negatives of a star were measured, the difference between the results was greater than the difference between two sets of measurements of one negative. In other words, the errors introduced by poor negatives slightly exceed the errors of measurement. It was only in the extreme cases of very dense or very thin negatives (particularly the former) that the effects were at all pronounced, and these were for the most part discarded. Errors due to these causes were minimised by the procedure, already mentioned, of measuring as many negatives as possible of each star.

In the second place, however, there are causes inherent in the stars themselves. Two stars of the same absolute magnitude may give spectra in which the lines differ in sharpness although presenting the same intensity. In such a case the wedge readings might be different, since it is found by experience that the sharpness or otherwise of a line affects its estimated intensity. The effect, however, is not so large as might be supposed, especially if both lines of a pair are similar in character. In any case the difficulty is not peculiar to the wedge method of measuring, since it is not easy by any method to compare the intensities of two lines, one of which is sharp and the other diffuse. The wedge will at least afford a mechanical (and consistent) means of avoiding any personal bias which might arise in the mere eye estimation of such cases.

Classification.

An accurate system of classifying stellar spectra by actual measurement of line intensities is very desirable as a preliminary step towards parallax work, in order to ensure that the groups of stars chosen shall be as homogeneous as possible as regards type. The system used at Harvard, based on eye estimations, is not entirely satisfactory. For example, there are gaps (such as that between B_5 and B_8) which appear to indicate, either that some stars from the types on each side of the gap are really of intermediate type, or that the steps between successive types are not truly uniform. The reduced line intensities, obtained as previously described, were therefore first investigated with a view to detecting lines which varied with spectral type. The Harvard (H.D.) classification was plotted against the intensities of suspected lines, and in

several cases some very promising results were obtained. This, of course, would be expected, since the Harvard classification is based (among other considerations) upon the successive development and strengthening or fading out of certain prominent lines. The actual measurement of the line intensities, however, gives more precision to the classification, and also tends to get rid of the gaps left in the Harvard sequence. In laying out the scale for spectral types these gaps were allowed for by leaving spaces to represent B_4, B_6, B_7, and so on, on the assumption that the Harvard observers were right in assigning a greater difference between the B_3 and B_5 types than between B_2 and B_3, and similarly for the other gaps. It is interesting to note that this assumption was justified by the absence of any discontinuity in the curves, and several stars were found whose classification had to be altered to one of the missing subgroups.

The lines which gave the best results for B stars were those of the diffuse series of the helium doublet system (λ4472 and λ4026). These gave well-defined curves from which spectral types could be accurately determined. The Mg^+ line at λ4481 also gave fairly good results when it was measurable, but the intensity in this case showed a maximum at about B_5, which rendered classification in that neighbourhood less accurate. It served as a check, however, and these three curves yielded classifications which in general were in good agreement, and the mean of which might be regarded as trustworthy.

The mean results given by these curves were generally not very different from the Harvard classification. The latter was consequently adopted in forming the three main groups (B_9 to A_8, B_0 to B_8, and O stars), each of which were treated as a whole in this preliminary work. Later investigations, however, will probably require the separate treatment of groups of stars within narrower limits of type, in order to increase the accuracy of the results. The measured classification just described will then be adopted.

Parallaxes.

The chief difficulty in applying this method to the determinations of parallaxes, lies in the lack of a sufficient number of reliable trigonometrical values upon which to found the curves. The majority of the B stars are so remote that they have not been included in the working lists of most observers; and in the comparatively few cases where observations have been made, the results are frequently either negative or smaller in magnitude than their probable errors. There are a few cases in which more or less trustworthy results have been obtained, but these are not sufficiently numerous to form a reliable basis for deriving others. It was therefore decided to use hypothetical parallaxes as well, since these are probably not affected by any very large systematic errors, and the effects of individual errors would be minimised by the comparatively large number of values available. For this purpose hypothetical parallaxes as computed from considerations of group motion and double-star orbits by Kapteyn,* Jackson and Furner,† and Plummer ‡

* *Astrophysical Journ.*, **40**, 43; *ibid.*, **47**, 255.
† *Monthly Notices, R.A.S.*, **81**, 2.
‡ *Monthly Notices, R.A.S.*, **72**, 170, 555; *ibid.*, **73**, 174, 450, 492.

were used, together with a few references to the work of other observers, such as Russell's dynamical parallaxes. Excessively large or small parallaxes (especially from Plummer's list) were not included, as being more likely to be erroneous; and where two or more values for one star were found, the arithmetical mean was adopted, except in the few cases

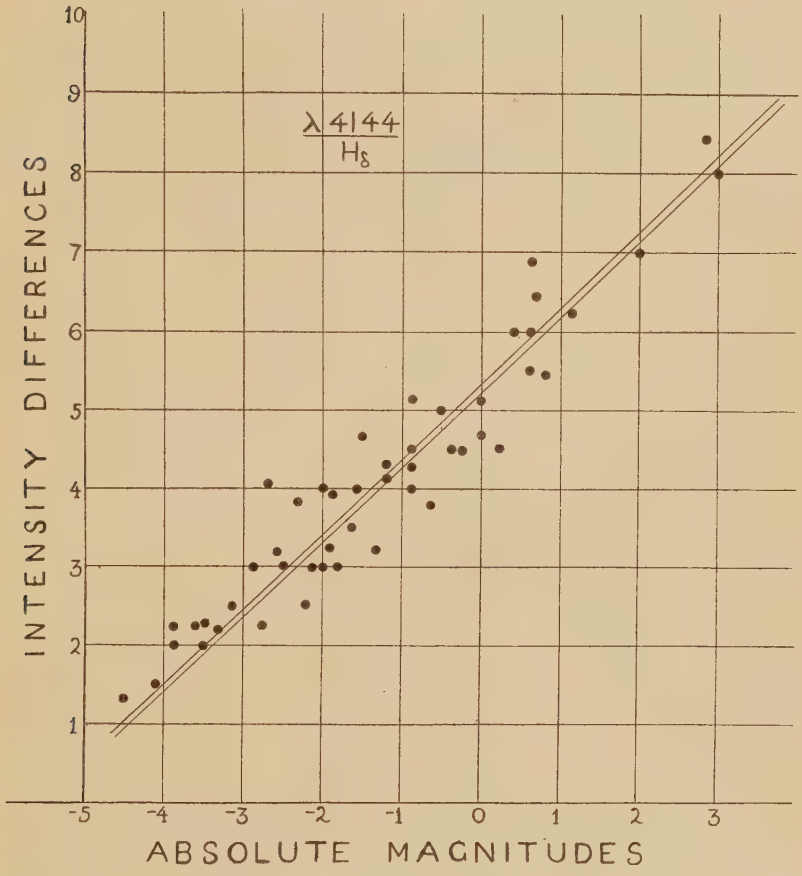


Fig. 1.

where the values were widely divergent. The four different methods served as a check on one another to some extent, and good general agreement between trigonometrical and hypothetical values seemed to indicate the absence of any large systematic error. In this way about sixty parallaxes of B stars were listed, of which fairly good negatives were available for measurement.

The preliminary measurements showed that the helium lines of the "diffuse singlet" series (*i.e.* "Parhelium" or "Asterium") were likely

to give very good results. The doublet series (λ4472, etc.) also showed a tendency to vary in intensity with absolute magnitude, but the points were too scattered to give a very reliable curve. In this case there was evidently a variation with type as well as with absolute magnitude

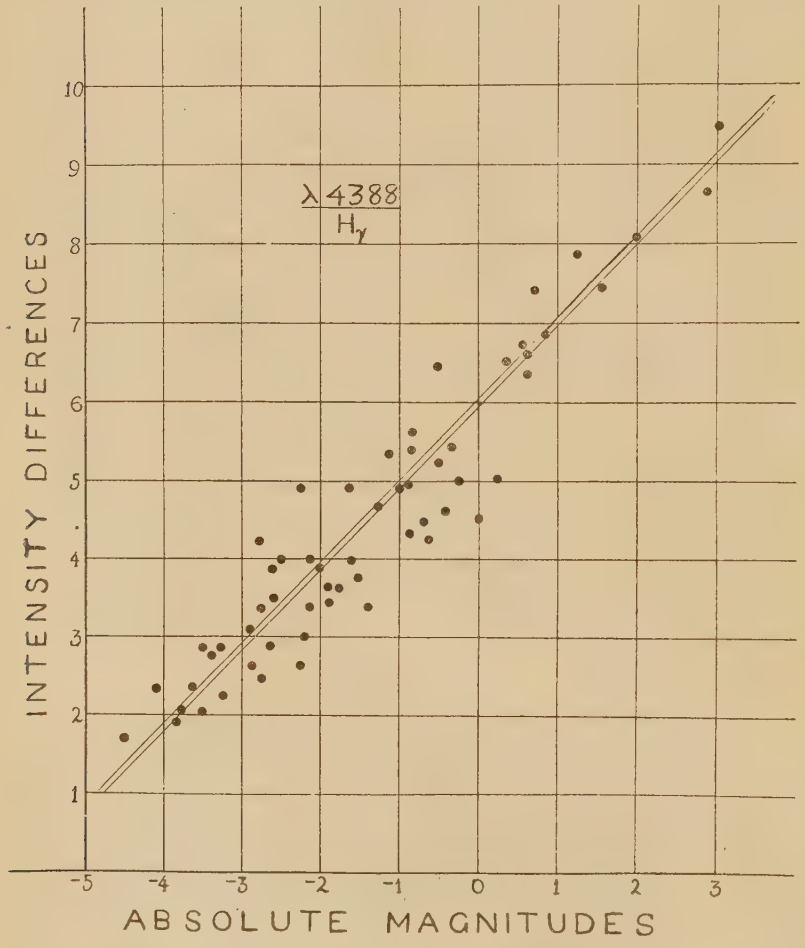


FIG. 2.

which complicated the curve. The two best lines in the former series (λ4388 and λ4144) were chosen, and compared directly with H_γ and H_δ respectively. The line at λ4009 is rather faint in many negatives, and was not used. It showed signs, however, of behaving in a similar manner to the other members of this series, and will probably be useful in a more complete investigation. The remaining strong line at λ4922 really requires panchromatic plates to give the best results, so

that attention was confined to the two pairs mentioned above. The resulting curves are shown in figs. 1 and 2.

The following table gives a representative selection of the results which have so far been obtained. The first four columns give the names, apparent magnitudes, and spectral types of the stars dealt with. The next four columns give the absolute magnitudes (M) and parallaxes (π) which were obtained from the two curves shown above, and column 9 gives the mean of these two parallaxes. This is compared in the last column with the adopted value derived from all known hypothetical and trigonometrical values.* It will be seen that there is fairly good agreement between the values derived from the two lines, except in the (intrinsically) very faint stars, in which the lines were rather difficult to measure. The means of these values (column 9), as will also be evident from an inspection of the curves, do not differ widely from the "observed" parallaxes.

TABLE I.

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
				λ4144.		λ4388.			
H.R.	Star.	Mag.	Spectr.	M.	π.	M.	π.	Mean Spectr. π.	Hyp. and Trig.
39	γ Peg.	2·9	B_2	−2·7	·008	−3·0	·007	·007	·008
130	κ Cas.	4·2	B_0	−2·0	·006	−2·4	·005	·005	
153	ζ Cas.	3·7	B_3	−2·3	·006	−2·5	·006	·006	·007
179	ξ Cas.	4·9	B_3	−1·1	·006	−1·8	·005	·005	
472	α Eri.	0·6	B_5	−0·8	·052	−1·0	·048	·050	·051*
542	ϵ Cas.	3·4	B_3	−1·5	·011	−2·5	·007	·009	·006
779	δ Cet.	4·0	B_2	−3·1	·004	−2·9	·004	·004	
801	35 Ari.	4·6	B_3	−2·4	·004	−2·4	·004	·004	
1123	40 Per.	5·0	B_2	−3·6	·002	−3·0	·003	·002	·002
1131	o Per.	3·9	B_1	−0·8	·012	−1·0	·011	·012	·019*
1142	17 Tau.	3·8	B_5	+1·7	·038	+2·0	·043	·040	·044
1145	19 Tau.	4·4	B_5	+0·3	·015	+0·6	·017	·016	·017
1149	20 Tau.	4·0	B_5	−0·2	·015	−0·3	·014	·015	·011
1156	23 Tau.	4·3	B_5	+1·7	·031	+0·7	·020	·025	·018
1165	η Tau.	3·0	B_{5p}	+1·1	·042	+1·5	·050	·046	·013
1173	27 Tau.	3·8	B_8	+3·0	·070	+2·5	·056	·063	·064
1203	ζ Per.	2·9	B_1	−2·3	·009	−2·4	·009	·009	·011
1320	μ Tau.	4·3	B_3	−0·3	·012	−0·7	·010	·011	·012
1350	53 Per.	4·9	B_3	−1·6	·005	−1·7	·005	·005	·008
1463	ν Eri.	4·1	B_2	−2·3	·005	−3·3	·003	·004	·004
1520	μ Eri.	4·2	B_5	−1·3	·008	−0·5	·012	·010	·010
1552	π^4 Ori.	3·8	B_3	−3·4	·004	−3·4	·004	·004	·005*
1765	22 Ori.	4·7	B_3	−2·3	·004	−2·3	·004	·004	·005

* Trigonometrical values are marked with an asterisk in last column.

TABLE I.—*continued.*

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.
H.R.	Star.	Mag.	Spectr.	λ4144. M.	λ4144. π.	λ4388. M.	λ4388. π.	Mean Spectr. π.	Hyp. and Trig.
1770	23 Ori.	5·0	B_3	−0·6	·008	−2·1	·004	·006	·005
1790	γ Ori.	1·7	B_2	−1·6	·023	−2·3	·016	·020	·018*
1839	32 Ori.	4·3	B_3	+1·2	·024	+0·4	·018	·021	·018
1934	ω Ori.	4·5	B_{3p}	−2·3	·004	−2·0	·005	·005	·005
1946	126 Tau.	4·9	B_3	−0·8	·007	−1·0	·007	·007	·008
2084	139 Tau.	4·9	B_2	−1·4	·005	−2·0	·004	·004	·005
2199	ξ Ori.	4·4	B_3	−1·2	·008	−0·6	·010	·009	·013
2294	β CMa.	2·0	B_1	−3·6	·008	−3·9	·007	·007	·007*
2618	ε CMa.	1·6	B_1	−4·5	·006	−4·0	·008	·007	·006
3454	η Hya.	4·3	B_3	−1·0	·009	−1·2	·008	·008	·008
4119	30 Sex.	5·0	B_5	+1·2	·017	+1·5	·020	·018	·013
4133	ρ Leo.	3·9	B_0	−2·2	·006	−2·7	·005	·005	
4199	θ Car.	3·0	B_0	−3·2	·006	−3·8	·004	·005	·005
4844	β Mus.	3·3	B_3	−2·3	·008	−2·6	·007	·008	·012
4853	β Cru.	1·5	B_1	−3·0	·012	−3·3	·011	·012	·018
5056	α Vir.	1·2	B_2	−2·0	·023	−1·5	·028	·025	·020*
5191	η UMa.	1·9	B_3	+0·7	·058	+0·5	·052	·056	·050
5576	κ Cen.	3·4	B_3	−1·8	·009	−1·0	·013	·011	·010
6462	γ Ara.	3·5	B_1	−4·1	·003	−3·5	·004	·004	·003
6527	λ Sco.	1·7	B_2	−3·3	·010	−3·0	·012	·011	·010
6580	κ Sco.	2·5	B_2	−3·2	·007	−3·5	·006	·007	·006
6714	67 Oph.	3·9	B_{5p}	−2·0	·007	−2·4	·005	·006	·005
7790	α Pav.	2·1	B_3	−2·4	·012	−2·4	·012	·012	
7852	ε Del.	4·0	B_5	−1·2	·009	−2·0	·006	·007	·010*
7963	λ Cyg.	4·5	B_5	+0·3	·015	+0·8	·018	·017	·020
8238	β Cep.	3·3	B_1	−3·0	·006	−3·7	·004	·005	·005

The accuracy of the results given in column 9 depends on two chief factors:—(1) the errors of measurement (including the effects of variations in negatives) which have already been mentioned, and which might cause probable errors of the order of ten per cent. of the parallax; and (2) errors in the fundamental parallaxes. The latter, unless systematic, would not be important, since they would merely effect a greater scattering of the points round the curves shown in figs. 1 and 2, and their effect would disappear in the mean. If there were any systematic error in these fundamental parallaxes, then of course the final results would be affected by an equal error. The best check on this source of error lies, as previously mentioned, in the agreement of the different

* Trigonometrical values are marked with an asterisk in last column.

hypothetical results among themselves, and with the trigonometrical results which are available.

There is also the possibility that the principles of the method itself are not rigidly correct. There may, for example, be other causes besides absolute magnitudes and spectral types which affect the intensities of lines. This consideration applies equally to all spectral types, and Professor Adams has shown that for lower-type stars the probable errors of the final results are about twenty per cent. of the parallax. It is therefore reasonable to suppose that in the case of B stars the probable errors do not exceed this amount, and may be less, since the errors decrease slightly with earlier types.

The parallaxes given in column 9 must not be regarded as final results, although they are probably very close approximations. It is hoped that a full list may shortly be published of all the stars measured in a slightly more complete investigation, in which the stars have been grouped within narrower limits of type and more lines have been utilised. The object of the present paper is to show that the method of spectroscopic parallax work can be applied to the B stars, and to indicate the nature of the results which may be expected.

Summary.

(1) An attempt has been made to apply the spectroscopic method of determining parallaxes to stars of higher spectral types than have hitherto been dealt with. Class B stars only are considered in this paper.

(2) A preliminary survey of suitable negatives was made in which the intensities of all measurable lines were referred to the average intensity of the hydrogen lines as a standard.

(3) These intensities were investigated with a view to determining systematic variations with spectral type or absolute magnitude. When a line was found to be variable it was remeasured in all negatives and compared with a neighbouring unaffected line instead of the standard hydrogen intensity.

(4) These pairs of lines gave curves showing the variation of their intensity with (*a*) Type or (*b*) absolute magnitude, by means of which further stars could be classified and their absolute magnitudes obtained.

(5) The lines used for classification were λ4472, λ4026, and λ4481. Those available for parallax were λ4388, λ4144, and (to a lesser extent) λ4009.

(6) The wedge method of measuring intensities was used throughout, and the various possible sources of error in connection with this are briefly considered.

(7) A preliminary list of results is given to show the possibility of obtaining parallaxes of B stars by this method and to indicate the probable accuracy of the results.

The Influence of Atmospheric Dispersion on Photographic Determinations of Stellar Parallax. By C. Davidson and W. M. H. Greaves.

(*Communicated by the Astronomer Royal.*)

The pioneer work in the modern photographic method of determining stellar parallax was done by Schlesinger * in 1903–1905 and by Russell and Hinks in 1903–1907.† A striking feature of this work was the care exercised in guarding against systematic error arising from various causes. Amongst errors of this nature are those which are usually grouped together in the category of hour-angle error. This category includes errors due to distortion arising from flexure and also errors arising from atmospheric dispersion. The effect of the latter is examined by Russell on pages 36–38 of the paper referred to above. He arrives at the conclusion that "even upon the most unfavourable hypothesis the average error of these parallaxes ‡ due to this cause would be only 0″·009."

Now for modern purposes a systematic error of such a magnitude would be inadmissible, and in fact its size was due to the fact that Russell used a non-neutral filter for the purpose of reducing the apparent photographic magnitude of the parallax star. In modern determinations this end is usually accomplished by the use of either a neutral filter or a rotating occulting shutter (or by a combination of these devices). The object of this note is to discuss the possible effect of atmospheric dispersion upon modern determinations of stellar parallax, and in particular upon the determinations made at the Royal Observatory, Greenwich.

Suppose the mean coefficient of refraction for the comparison stars is β, and the coefficient of refraction for the parallax star is $\beta+\delta\beta$.

Then the parallax star will be displaced relatively to the comparison stars by an amount $X\delta\beta$, where X is the x co-ordinate of the zenith on the plate. If this displacement is not allowed for it will be interpreted as a parallactic displacement, and a corresponding error will appear in the value of the derived parallax.

Suppose the hour angle west of the plate centre at the middle of the exposure be τ. Let λ be the co-latitude, and P the north polar distance of the plate centre. Then

$$X = \frac{\tan\tau \sin q}{\cos(P-q)} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where

$$\tan q = \tan\lambda \cos\tau$$

* *A.P.J.*, 20, 123, 1904, and subsequent papers.

† H. N. Russell, *Determinations of Stellar Parallax*, Publication No. 147 of the Carnegie Institute of Washington.

‡ *I.e.* of the parallaxes determined by Russell and Hinks.

In parallax work the photographs are taken on or near the meridian, so that τ is small. Neglecting its square, we have

$$q = \lambda,$$

and then

$$X = \cdot 0044 \sec z \,.\, \sin \lambda \,.\, \tau \quad . \quad . \quad . \quad . \quad (2)$$

where τ is measured in minutes of time, and $z = P - \lambda$. (When the plate-centre is on the meridian, z is its zenith distance south.)

So that the displacement of the parallax star relatively to the comparison stars is

$$n\tau \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where

$$n = \cdot 0044 \sec z \sin \lambda \delta\beta \quad . \quad . \quad . \quad . \quad (4)$$

for Greenwich $\lambda = 38^\circ\ 31'$, giving $n = \cdot 0027 \sec z \delta\beta$.

In practice a series of plates is taken, and one plate of the series is selected as a scale, the others being compared with it in turn. Let τ_0 be the hour angle west at which the scale plate was taken. Then on comparing a plate taken at hour angle τ with the scale we should have a residual $n(\tau - \tau_0)$ for the parallax star due to atmospheric dispersion.

Let R denote the total residual for the plate in question. The parallax and proper motion are determined from a least square solution of the equations

$$f\pi + t\mu + c = R \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where π is the parallax, μ the proper motion, f and t the parallax factor and time factor respectively, and c is a constant.

But the values of π and μ as obtained from equations (5) will be subject to error, owing to the fact that the residuals R are not the true residuals due to proper motion and parallax, but contain portions due to the effect of atmospheric dispersion. The corrected values π_1 and μ_1 of the parallax and proper motion will be obtained by a least square solution of the equations

$$f\pi_1 + t\mu_1 + c = R - n(\tau - \tau_0),$$

or writing

$$c - n\tau_0 = c_1,$$

$$f\pi_1 + t\mu_1 + c_1 = R - n\tau \quad . \quad . \quad . \quad . \quad (6)$$

The solution will be of the form

$$\left.\begin{aligned} \pi_1 &= \pi + hn \\ \mu_1 &= \mu + kn \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where π and μ are the uncorrected values of the parallax and proper motion as derived from (5), and h and k are numerical coefficients.

If the equations (5) have been already solved, the extra work necessary for the determination of h and k is comparatively small.

We may write $\pi_1 = \pi + \delta\pi$, $\mu_1 = \mu + \delta\mu$, so that

$$\left.\begin{aligned} \delta\pi &= hn \\ \delta\mu &= kn \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

We thus get the corrections $\delta\pi$ and $\delta\mu$, which must be added to the uncorrected values π and μ of the parallax and proper motion in order to correct for atmospheric dispersion. So far we have not put in the numerical value of n. It seems advisable to defer this as long as possible, owing to uncertainty as to the effective photographic wave-lengths of the individual stars.

As an example the star Oeltz N 15254 (B.D. 72°674) may be considered. For this star the values π and μ of the parallax and proper motion, no allowance being made for atmospheric dispersion, have been determined at Greenwich, and are

$$\pi = +0''\cdot011 \pm 0\cdot010$$
$$\mu = -0''\cdot188 \pm 0\cdot003.$$

The values of $\delta\pi$ and $\delta\mu$, determined in the above manner, are

$$\delta\pi = +11\cdot0n$$
$$\delta\mu = +3\cdot8n.$$

The following table, applicable to determinations made with reflecting telescopes, gives the assumed effective photographic wave-lengths and the corresponding values of $\delta\beta$ for stars of different spectral types.

TABLE I.

Reflecting Telescopes.

Spectral Type of Parallax Star.	Assumed Effective Photographic Wave-length.	$\delta\beta$.
B_0	4100	+0''·22
B_5	4170	+0·16
A_0	4260	+0·09
A_5	4270	+0·08
F_0	4280	+0·07
F_5	4290	+0·06
G_0	4310	+0·05
G_5	4390	−0·02
K_0	4470	−0·08
K_5	4540	−0·14
M	4520	−0·12

The figures in the second column were derived from the results obtained at Greenwich with the 30-in. reflector.* It was assumed that the mean effective photographic wave-length of the comparison stars in a parallax determination is $\lambda = 4370$. From the experimental results of Kayser and Runge† the value 0''·08 per 100 λ was adopted as the

* C. Davidson and E. Martin, "Effective Photographic Wave-lengths of Stars in Relation to Spectral Type and Colour Index," *Monthly Notices, R.A.S.*, **82**, 2, 66–77, 1921 December.

† *Wied. Ann.*, 50, 293, 1893.

rate of change of β between $\lambda = 4100$ and $\lambda = 4500$, and the third column of the table was then computed.

In the case of refracting telescopes, where the effective length of the spectrum is shortened owing to the increased absorption by the objective in the violet, the values of $\delta\beta$ will be smaller. For the Greenwich 26-in. refractor a preliminary investigation has shown that $\delta\beta$ will be reduced to about $\frac{6}{10}$ of the above values. For observations taken with visual telescopes using a yellow filter, the values of $\delta\beta$ will be presumably still less.

For the star Oeltz Arg. N 15254, cited above, the spectral type is K_0. The table then gives $\delta\beta = -0\cdot08$. Multiplying by ·6 (since the Greenwich parallaxes are determined with the 26-in. refractor) we get $\delta\beta = -0''\cdot05$. We also have $z = -20^\circ\ 34'$, and we obtain $n = -\cdot00014$, and substituting in the expressions obtained above for $\delta\pi$ and $\delta\mu$ we get

$$\delta\pi = -''\cdot0015$$
$$\delta\mu = -\ \cdot0005.$$

An approximate value of $\delta\pi$ may be obtained very quickly in the following way:—

Let τ_m and τ_e denote the mean hour angles for the morning and evening observations, and f_m and f_e the mean parallax factors.

Then we have approximately

$$\delta\pi = -\frac{\tau_m - \tau_e}{f_m - f_e} n \qquad . \quad . \quad . \quad . \quad (9)$$

For Oeltz Arg. N 15254, (9) gives $\delta\pi = 10n = -\cdot0014$.

Let us suppose that $\tau_m - \tau_e = 20$ minutes, which represents a rather exceptional range of hour angle, and $z = 30^\circ$. We may take $f_m - f_e = 1\cdot5$, which is in most cases roughly correct. The values then obtained for $\delta\pi$ for stars of different spectral types are given in the following table:—

TABLE II.

Approximate Values of $\delta\pi$ (*Reflecting Telescopes*) *for* $\tau_m - \tau_e = 20^m$, $f_m - f_e = 1\cdot5$, $\lambda = 38^\circ\ 31'$ and $z = 30^\circ$.

Spectral Type of Parallax Star.	$\delta\pi$.
B_0	$-0''\cdot0091$
B_5	$-0''\cdot0066$
A_0	$-0''\cdot0037$
A_5	$-0''\cdot0033$
F_0	$-0''\cdot0029$
F_5	$-0''\cdot0025$
G_0	$-0''\cdot0021$
G_5	$+0''\cdot0008$
K_0	$+0''\cdot0033$
K_5	$+0''\cdot0058$
M	$+0''\cdot0050$

Table II. was computed from the values of $\delta\beta$ given in Table I. It should give an idea of the magnitude of $\delta\pi$ for stars of different spectral type. The corresponding values of $\delta\pi$ for refracting telescopes may be obtained by multiplying the values given in Table II. by $\frac{6}{10}$.

It is clear from the foregoing that with the standard of accuracy aimed at in modern parallactic determinations, the error arising from atmospheric dispersion cannot be ignored. At the same time, owing to the uncertainty of the effective photographic wave-length of the parallax star, no system of correction is entirely satisfactory. This suggests the desirability of reducing the error by using as small a range of hour angle as possible, especially in the case of stars of late or early spectral type, for which the values of $\delta\pi$ are comparatively large. For such stars the importance of working on the meridian as closely as possible is emphasised.

Note.—The whole effect of hour-angle error (which includes the error due to atmospheric dispersion) may be allowed for by introducing an extra term into the equations of condition. Thus, instead of solving the equations

$$f\pi + t\mu + c = \mathrm{R}$$

by least squares, we may solve the equations

$$f\pi + t\mu + c + \tau n = \mathrm{R}$$

for the unknowns, π, μ, c, and n, n being an (unknown) coefficient of hour-angle error.

For the star Oeltz Arg. N 15254 this method gives

$$n = -0''{\cdot}0007 \pm ''{\cdot}0007,$$

corresponding to $\delta\pi = -0''{\cdot}008 \pm ''{\cdot}008$.

The Effect of the Second-Order Terms in Differential Aberration on Determinations of Stellar Parallax. By W. M. H. Greaves.

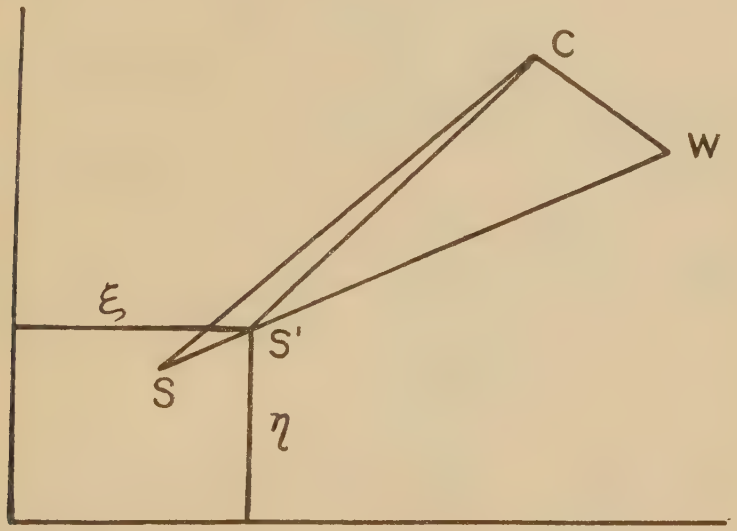
(*Communicated by the Astronomer Royal.*)

In determinations of stellar parallax the effect of the first-order terms in differential refraction and differential aberration are included in the "plate constants" in the usual way, it being tacitly assumed that the effect of the second-order terms is negligible. It has been pointed out by Russell* that, as far as differential refraction is concerned, for plates having the same centre among the stars and taken at an (approximately) fixed hour angle,† the non-linear terms are practically constant for each star, and affect the star places alone, not the parallax.

* Russell, *Determinations of Stellar Parallax*, ch. 2, p. 19 (Publication No. 147 of the Carnegie Institute of Washington, 1911).

† This condition is fulfilled in parallactic determinations.

The object of this note is to investigate the effect of the second-order terms arising from differential aberration. It will be shown that, owing to the fact that the comparison stars are usually so chosen that they lie roughly on the circumference of a circle whose centre is the parallax star, the omission of the second-order terms will lead to a systematic error in the derived parallax, but that this error is so small that with the present standard of accuracy it may be neglected.



Let S′ be a star image on a parallax plate, S its position corrected for aberration, W the projection on the plate of the apex of the earth's way.

Let C be the centre of projection (the optical centre of the object glass).

Then

$$\sin \mathrm{SCS}' = \gamma_0 \sin \mathrm{S}'\mathrm{CW} \quad . \quad . \quad . \quad . \quad (1)$$

where γ_0 is the constant of aberration.

Let the co-ordinates on the plate of S′, S, and W be (ξ, η), $(\xi - d\xi, \eta - d\eta)$, and (XY) respectively.

Now

$$\begin{aligned}\frac{\mathrm{SS}'}{\mathrm{S}'\mathrm{W}} &= \frac{\mathrm{SS}'}{\mathrm{S}'\mathrm{C}} \cdot \frac{\mathrm{S}'\mathrm{C}}{\mathrm{S}'\mathrm{W}} \\ &= \frac{\sin \mathrm{SCS}'}{\sin \mathrm{CSW}} \cdot \frac{\sin \mathrm{CWS}}{\sin \mathrm{S}'\mathrm{CW}} \\ &= \gamma_0 \frac{\sin \mathrm{CWS}}{\sin \mathrm{CSW}} \quad \text{from (1)} \\ &= \gamma_0 \frac{\mathrm{CS}}{\mathrm{CW}},\end{aligned}$$

and therefore,

$$\left.\begin{aligned} d\xi &= \frac{SS'(X-\xi)}{S'W} = \gamma_0 \frac{CS}{CW}(X-\xi) \\ d\eta &= \frac{SS'(Y-\eta)}{S'W} = \gamma_0 \frac{CS}{CW}(Y-\eta) \end{aligned}\right\} \quad . \quad . \quad . \quad (2)$$

But

$$\frac{CS}{CW} = \frac{\sqrt{1+(\xi-d\xi)^2+(\eta-d\eta)^2}}{\sqrt{1+X^2+Y^2}},$$

the unit of length being taken to be the distance between the optical centre of the object-glass and the plate centre.

We shall neglect terms of order $\gamma_0\xi^3$, of order $\gamma_0\xi d\xi$, and of order $\gamma_0 d\xi^2$.

Then

$$\frac{CS}{CW} = \frac{1}{\sqrt{1+X^2+Y^2}}\{1+\tfrac{1}{2}(\xi^2+\eta^2)\}.$$

Hence

$$d\xi = \frac{\gamma_0 X}{\sqrt{1+X^2+Y^2}}\left\{1-\frac{\xi}{X}\right\}\{1+\tfrac{1}{2}(\xi^2+\eta^2)\}$$

$$d\eta = \frac{\gamma_0 Y}{\sqrt{1+X^2+Y^2}}\left\{1-\frac{\eta}{Y}\right\}\{1+\tfrac{1}{2}(\xi^2+\eta^2)\},$$

giving

$$\left.\begin{aligned} d\xi &= \frac{\gamma_0 X}{\sqrt{1+X^2+Y^2}} - \frac{\gamma_0\xi}{\sqrt{1+X^2+Y^2}} + \tfrac{1}{2}\gamma_0 \frac{X}{\sqrt{1+X^2+Y^2}}\{\xi^2+\eta^2\} \\ d\eta &= \frac{\gamma_0 Y}{\sqrt{1+X^2+Y^2}} - \frac{\gamma_0\eta}{\sqrt{1+X^2+Y^2}} + \tfrac{1}{2}\gamma_0 \frac{Y}{\sqrt{1+X^2+Y^2}}\{\xi^2+\eta^2\} \end{aligned}\right\} \quad (3)$$

In the expressions (3) for $d\xi$ and $d\eta$ the terms not involving ξ and η and the first-order terms in ξ and η enter into the plate constants in the usual way. They are the terms which have been given by Turner.*

There remain the second-order terms, which we shall denote by $d\xi_2$ and $d\eta_2$, so that

$$\left.\begin{aligned} d\xi_2 &= \tfrac{1}{2}\gamma_0 \frac{X}{\sqrt{1+X^2+Y^2}}(\xi^2+\eta^2) \\ d\eta_2 &= \tfrac{1}{2}\gamma_0 \frac{Y}{\sqrt{1+X^2+Y^2}}(\xi^2+\eta^2) \end{aligned}\right\} \quad . \quad . \quad . \quad (4)$$

We notice that $d\xi_2$ and $d\eta_2$ are constant for stars lying on a circle whose centre is the plate centre. This condition is, as a rule, roughly fulfilled by the comparison stars, the parallax star being usually close to the plate centre.

Parallax determinations are usually made by an accurate measurement of the x co-ordinates, so we shall only consider $d\xi_2$.

* H. H. Turner, *Monthly Notices R.A.S.*, **54**, 1, 20, Nov. 1893.

Suppose the parallax star is at the plate centre and the comparison stars lie on a circle whose centre is the plate centre and whose radius is ρ. Then we have, for each comparison star,

$$d\xi_2 = \tfrac{1}{2}\gamma_0 \frac{X\rho^2}{\sqrt{1+X^2+Y^2}},$$

and for the parallax star

$$d\xi_2 = 0,$$

so that the parallax star is displaced relatively to the comparison stars in the positive direction of the x axis by an amount

$$\frac{-\tfrac{1}{2}\gamma_0 X\rho^2}{\sqrt{1+X^2+Y^2}}.$$

If this displacement is not allowed for, it will be interpreted as a parallactic displacement.

Let us consider the ideal case of a parallactic determination from two plates. Let $(X_1 Y_1)$, $(X_2 Y_2)$ be the corresponding co-ordinates of the apex, f_1 and f_2 the parallax factors, and D_1 and D_2 the relative displacements of the parallax star.

Then

$$D_1 = -\tfrac{1}{2}\gamma_0 \frac{X_1\rho^2}{\sqrt{1+X_1^2+Y_1^2}}$$

$$D_2 = -\tfrac{1}{2}\gamma_0 \frac{X_2\rho^2}{\sqrt{1+X_2^2+Y_2^2}},$$

and if $\delta\pi$ is the correction for aberration which must be added to the derived parallax,

$$\delta\pi = -\frac{D_1 - D_2}{f_1 - f_2} = \tfrac{1}{2}\frac{\gamma_0\rho^2}{f_1 - f_2}\left(\frac{X_1}{\sqrt{1+X_1^2+Y_1^2}} - \frac{X_2}{\sqrt{1+X_2^2+Y_2^2}}\right).$$

Without going into detail any further we can get an upper bound for $\delta\pi$. Since

$$\frac{X}{\sqrt{1+X^2+Y^2}} < 1 \text{ numerically we have}$$

$$\delta\pi < \frac{\gamma_0\rho^2}{f_1 - f_2} \text{ numerically.}$$

Taking $\rho = 20'$, $f_1 - f_2 = 1{\cdot}5$, and using the value $\gamma_0 = 20''{\cdot}4$, we get

$$\delta\pi < 0''{\cdot}0005,$$

indicating that with the present standard of accuracy in parallactic determinations the error due to the neglect of the second-order terms in differential aberration may be disregarded.

Stellar Parallaxes determined at the Royal Observatory, Greenwich.

(*Communicated by the Astronomer Royal.*)

The following list is a continuation of the results obtained with the 26-inch telescope.

In the column Proper Motion the first is derived from meridian observations; the second, in R.A. only, is the relative proper motion derived from the parallax plates. The motions in both R.A. and Dec. are expressed in arc of a great circle.

In calculating the parallax, the relative proper motion is used, *not* the meridian proper motion.

The Probable Error has been calculated by the formula

$$r = 0{\cdot}6745 \times \sqrt{\frac{[vv]}{m-\mu}} \times \frac{1}{\sqrt{w}}.$$

The first given is the P.E. of the parallax, the second is the P.E. of the relative proper motion.

Generally six comparison stars of mean magnitude 11–12 have been used, the magnitude of the parallax star being reduced to equality by means of the rotating sector.

No.	Name and B.D. No.	Mag. Vis. Photog.	Spectral Type.	R.A. Dec. 1900·0.	Proper Motion. R.A.	Proper Motion. Dec.	Parallax.	P.E.	No. of Plates.
		m		h m s	″	″	″	″	
1	L.F. 4660	9·0	Fo	0 2 14	+ ·337	− ·024	+ ·003	± ·008	16
	85° 412	8·5		+86° 14′	+ ·341			± ·002	
2	L. 660	7·2	F8	0 25 16	+ ·299	− ·084	+ ·028	± ·008	16
	68° 29	7·5		+69° 14′	+ ·302			± ·002	
3	Gr. 307	7·5	G5	1 21 2	+ ·203	− ·140	+ ·016	± ·008	19
	73° 75	7·6		+73° 41′	+ ·188			± ·003	
4	L. 2751	6·8	G5	1 27 21	− ·395	+ ·131	+ ·035	± ·006	24
	68° 113	6·9		+68° 26′	− ·382			± ·002	
5	69° 253	9·4		4 15 44	+ ·144	− ·162	+ ·018	± ·007	19
		10·1		+69° 52′	+ ·128			± ·007	
6	88° 25	9·5		4 59 41	+ ·166	− ·186	+ ·027	± ·005	18
		10·7		+88° 5′	+ ·165			± ·006	
7	O.A. 6778	9·3		6 17 55	+ ·059	− ·213	+ ·026	± ·006	22
	68° 441	9·7		+68° 15′	+ ·055			± ·003	
8	74° 290	8·8		6 23 4	− ·032	− ·197	+ ·013	± ·006	21
		9·0		+74° 22′	− ·055			± ·006	
9	72° 322	7·5	Go	6 24 36	− ·014	− ·220	+ ·041	± ·007	19
		8·0		+72° 5′	− ·023			± ·007	
10	71° 380	9·4		6 52 36	− ·029	− ·476	+ ·018	± ·006	21
		9·5		+71° 21′	− ·034			± ·007	

No.	Name and B.D. No.	Mag. Vis. Photog.	Spectral Type.	R.A. Dec. 1900·0.	Proper Motion. R.A.	Proper Motion. Dec.	Parallax.	P.E.	No. of Plates.
		m		h m s	″	″	″	″	
11	K. 1333	9·4		7 14 23	+ ·160	− ·280	+ ·034	± ·008	20
		10·6		+76° 19′	+ ·224			± ·009	
12	O.A. 8433–4	8·9		7 52 27	+ ·022	− ·186	+ ·012	± ·006	19
	69° 452	9·2		+69° 39′	+ ·064			± ·007	
13	67° 552	9·3		8 27 23	− 1·101	− ·048	+ ·067	± ·006	16
		9·5		+67° 38′	− 1·052			± ·007	
14	O.A. 11677	9·0		11 14 53	− 2·957	+ ·203	+ ·120	± ·010	18
	66° 717	10·3		+66° 23′	− 2·936			± ·010	
15	79° 379	9·4		11 52 10	− ·206	− ·010	+ ·018	± ·005	20
		10·1		+78° 57′	− ·215			± ·004	
16	73° 566	9·3		12 38 17	− ·315	− ·056	+ ·012	± ·007	22
		9·4		+73° 31′	− ·291			± ·005	
17	71° 646	9·5		13 11 45	− ·158	− ·050	+ ·017	± ·008	23
		10·2		+70° 50′	− ·145			± ·006	
18	75° 511	9·5		13 33 14	− ·461	− ·012	+ ·039	± ·007	21
		10·5		+75° 1′	− ·406			± ·004	
19	77° 521	9·3		13 43 49	− ·156	− ·046	+ ·026	± ·007	19
		9·8		+77° 44′	− ·139			± ·005	
20	81° 465	9·5		14 10 20	+ ·184	− ·549	+ ·058	± ·008	24
		11·0		+81° 5′	+ ·224			± ·005	
21	75° 551	9·4		15 2 8	− ·301	− ·092	+ ·028	± ·008	24
		10·2		+75° 39′	− ·263			± ·006	
22	L.F. 2726	7·0	G5	15 58 51	− ·058	+ ·271	+ ·035	± ·008	23
	71° 762	7·7		+71° 10′	− ·062			± ·003	
23	69° 839	9·3		16 13 20	+ ·033	− ·212	+ ·011	± ·006	24
		11·3		+69° 30′	− ·006			± ·003	
24	P. XVI. 195	6·5	G5	16 34 56	− ·093	+ ·271	+ ·008	± ·007	10
	77° 627	7·4		+77° 39′	− ·086			± ·008	
25	75° 606	9·2		16 54 53	+ ·092	− ·235	+ ·022	± ·007	27
		9·6		+75° 53′	+ ·102			± ·005	
26	h^1 Dra.	5·3	F5	16 55 29	+ ·247	+ ·049	+ ·026	± ·006	10
	65° 1157	5·4		+65° 17′	+ ·213			± ·016	
27	86° 250	9·5		16 58 24	− ·186	+ ·145	+ ·016	± ·008	26
		10·5		+86° 4′	− ·167			± ·005	
28	69° 940	9·5		17 41 46	+ ·130	+ ·179	+ ·012	± ·009	21
		10·0		+69° 16′	+ ·161			± ·005	
29	70° 950	9·4		17 42 40	− 196	+ ·065	+ ·029	± ·007	27
		10·0		+70° 30′	− ·200			± 004	

No.	Name and B.D. No.	Mag. Vis. Photog.	Spectral Type.	R.A. Dec. 1900·0.	Proper Motion. R.A.	Proper Motion. Dec.	Parallax.	P.E.	No. of Plates.
		m		h m s	″	″	″	″	
30	L. 32769	8·2	Ko	17 43 3	− ·127	+ ·320	+ ·048	± ·010	23
	72° 803	8·1		+72° 27′	− ·090			± ·005	
31	67° 1034	9·2		17 44 47	− ·116	− ·247	+ ·004	± ·008	26
		9·8		+67° 20′	− ·128			± ·004	
32	81° 623	9·5		18 27 42	+ ·042	+ ·212	+ ·015	± ·011	21
		11·1		+81° 10′	+ ·035			± ·007	
33	81° 640	9·3		18 45 10	− ·028	+ ·305	+ ·017	± ·006	25
		10·1		+81° 16′	− ·038			± ·003	
34	68° 1027	9·1		18 49 17	+ ·108	+ ·200	+ ·012	± ·008	24
		9·6		+68° 45′	+ ·072			± ·004	
35	82° 565	9·3		18 58 35	+ ·239	+ ·281	− ·001	± ·008	25
		9·4		+82° 35′	+ ·205			± ·004	
36	69° 1043	9·0		19 20 52	+ ·095	+ ·226	+ ·008	± ·010	17
		9·6		+69° 43′	+ ·085			± ·008	
37	76° 735	9·4		19 25 35	+ ·182	+ ·370	+ ·029	± ·007	22
		10·2		+76° 55′	+ ·149			± ·004	
38	σ Dra.	5·0	Ko	19 32 33	+ ·555	+ 1·754	+ ·165	± ·015	16
	69° 1053	5·4		+69° 30′	+ ·582			± ·033	
39	L. 37923	8·4	Ko	19 42 11	+ ·172	+ ·105	+ ·060	± ·006	14
	76° 750	8·7		+76° 11′	+ ·137			± ·013	
40	O.A. 20166	8·0	G	20 5 46	+ ·134	+ ·166	− ·001	± ·012	14
	72° 933	9·0		+72° 42′	+ ·192			± ·029	
41	70° 1112	9·2		20 14 44	+ ·024	− ·255	+ ·014	± ·006	25
		9·5		+70° 32′	+ ·046			± ·004	
42	83° 582	9·4		20 31 44	+ ·162	+ ·290	+ ·009	± ·009	24
		10·4		+83° 40′	+ ·172			± ·006	
43	L. 40215	7·2	G5	20 38 42	+ ·368	+ ·516	+ ·038	± ·010	21
	75° 752	8·1		+75° 14′	+ ·372			± ·003	
44	O.A. 22353	8·8		21 25 42	+ ·221	− ·184	+ ·012	± ·007	21
	63° 1737	9·2		+63° 56′	+ ·215			± ·002	
45	73° 1051	8·7	G5	23 36 45	− ·220	+ ·045	+ ·014	± ·009	17
		9·2		+73° 51′	− ·234			± ·003	

Notes.

No. 14. O.A. 11677. Spectroscopic Par. ″·219.
No. 26. h^1 Draconis. Allegh. +″·082 ±″·007.
Spectro. ·058.

The Greenwich separate results for h^1 Draconis are accordant, but the star is kept on the observing list for further observations.

Fourteenth Note on the Number of Faint Stars with Large Proper Motions on plates of the Oxford Astrographic Catalogue for Zones $+25°$, $+26°$, and $+27°$. By F. A. Bellamy, M.A.

The previous notes of this series are in *M.N.*, **69**, 57, 491 ; **71**, 45 582 ; **72**, 65 ; **74**, 26, 27 ; **76**, 102, 428, 538, 657 ; **78**, 471, 591. The present note deals with ten pairs of plates, measured in the first instance at different times, but in all cases of suspected P.M. remeasured recently by myself, as also were similar cases on the modern plates.

The method of procedure is fully explained in former notes.

Table I. gives a general summary of the results obtained, and Table II. the details.

The region most bare of stars down to the eleventh magnitude in the whole of the Oxford zones $+25°$ to $+31°$ is partly included in $+27°$, $4^h\ 20^m$; 37 stars in a square degree is the representative density over a considerable area.

TABLE I.

Stars with Probable Large Proper Motions: Summary.

Zone. R.A. Plate Centre.	Limiting Dates.	Interval in Years.	Total Stars compared.	Centennial P.M.'s. Above 15″.	Below 15″.
° h m					
+26 0 0	1895–1915	20·02	179	3	10
+26 2 56	1892–1916	23·15	199	3	7
+27 4 20	1895–1907	12·10	15	2	0
+25 12 28	1893–1916	23·01	70	1	8
+25 13 16	1893–1916	23·01	72	2	8
+25 13 24	1893–1916	23·00	49	2	7
+25 13 40	1893–1916	23·01	95	1	8
+25 13 48	1893–1916	23·03	85	4	15
+25 13 56	1893–1916	23·03	77	3	9
+26 14 48	1895–1916	20·67	164	2	12
	Totals		1005	23	84

Six of the regions in Table I. have already been examined after an interval of 16·7 years (see the Fifth Note of this series), when they yielded 12 stars with centennial P.M. $>15''$, and 26 between $15''$ and $8''·9$. Naturally with the longer interval of twenty-three years, a larger number of stars show the standard displacement (given by $\Delta x^2 + \Delta y^2 = 25$), and we now get 55 stars passing this test and yielding C.P.M.'s. down to $6''·3$. It is satisfactory that the longer interval (allowing greater precision) has not diminished but increased the number $>15''$ from 12 to 13.

TABLE II.

List of Large Proper Motions (Zones 25° to 27°).

Ref. No.	Plate Centre.			Oxford.		A.G.C. No.	Δx. (Unit = 0″·3.)	Δy.	Cent. P.M. in Arc.	Interval in Years.
	°	h	m	No.	φ Mag.					
1	+26	0	0	132*	6·7	C 14413	+52	−65	124″·7	20·02
2	+26	0	0	41	11·4		−22	−41	69·8	20·02
3	+25	13	56	42474	10·1		−40	− 6	52·7	23·03
4	+25	12	28	40829	10·5		−31	− 3	40·6	23·01
5	+26	14	48	36079*	10·1	C 7013	−13	−20	34·6	20·67
6	+26	2	56	7181	10·8		+17	−17	31·2	23·15
7	+26	14	48	36086*	9·5	C 6981	0	−21	30·5	20·67
8	+25	13	16	41942*	8·1	C 6430	−19	+ 8	26·9	23·01
9	+26	2	56	7233*	7·6	C 1553	+17	−10	25·6	23·15
10	+25	13	40	42234	10·6		−13	+10	21·4	23·01
11	+25	13	24	42027*	6·3	C 6514	+ 2	−16	21·1	23·00
12	+25	13	56	42461*	9·1	C 6659	+ 1	−16	20·9	23·03
13	+25	13	48	42366	10·4		−15	+ 4	20·2	23·03
14	+25	13	24	42004*	8·4	B 4783	−15	+ 3	20·0	23·00
15	+25	13	48	42329	10·1		−13	+ 8	19·9	23·03
16	+25	13	48	42367*	9·4	C 6659	+ 5	−14	19·4	23·03
17	+25	13	56	42441*	8·3	C 6651	−13	+ 5	18·2	23·03
18	+27	4	20	9648*	8·0	C 2065	+ 4	− 6	17·9	12·10
19	+26	2	56	7247*	9·2	C 1530	0	+13	16·9	23·15
20	+25	13	48	42350*	9·0	C 6651	−12	+ 4	16·5	23·03
21	+26	0	0	84*	7·6	C 12	+ 9	− 6	16·0	20·02
22	+27	4	20	9647*	6·1	C 2094	+ 4	+ 5	15·9	12·10
23	+25	13	16	41931	11·4		−11	+ 4	15·3	23·01
24	+26	0	0	166	8·9		+ 8	− 1	12·2	20·02
25	+25	13	48	42373*	7·6	C 6624	− 6	+ 7	12·0	23·03
26	+26	14	48	36017*	9·0	C 7010	− 8	+ 2	12·0	20·67
27	+25	12	28	40873	10·8		− 1	− 9	11·8	23·01
28	+25	13	56	42478*	8·9	C 6688	+ 1	+ 9	11·8	23·03
29	+26	14	48	36098	10·9		− 4	+ 7	11·7	20·67
30	+25	12	28	40891	9·2		0	− 9	11·7	23·01
31	+26	0	0	50*	8·7	C 25	− 6	− 5	11·6	20·02
32	+26	14	48	36060	10·7		0	− 8	11·6	20·67
33	+26	14	48	36140	10·4		+ 6	− 5	11·3	20·67
34	+25	13	40	42232	10·7		− 5	+ 7	11·2	23·01
35	+25	12	28	40887	10·5		− 6	+ 6	11·1	23·01
36	+25	13	16	41907*	9·9	B 4763	− 6	+ 6	11·1	23·01
37	+25	13	48	42359	9·8		− 2	+ 8	10·8	23·03
38	+25	13	40	42213	10·3		+ 8	+ 2	10·7	23·01

TABLE II.—*continued.*

Ref. No.	Plate Centre.	Oxford. No.	Oxford. φ Mag.	A.G.C. No.	Δx. (Unit=0″·3.)	Δy. (Unit=0″·3.)	Cent. P.M. in Arc.	Interval in Years.
	° h m						″	
39	+26 0 0	24	9·5		− 5	− 4	9·7	20·02
40	+25 13 24	42037*	9·6	C 6474	− 7	+ 2	9·5	23·00
41	+25 13 56	42408	9·7		− 2	− 7	9·5	23·03
42	+25 13 40	42226	10·6		− 7	+ 2	9·5	23·01
43	+26 0 0	83	11·2		+ 6	− 3	9·5	20·02
44	+26 2 56	7179	10·1		− 7	− 2	9·4	23·15
45	+26 2 56	7201	11·7		− 2	− 7	9·4	23·15
46	+26 0 0	37	9·9		+ 6	+ 2	9·3	20·02
47	+26 0 0	176	10·8		− 6	− 2	9·2	20·02
48	+25 13 16	41957	11·7		+ 7	0	9·1	23·01
49	+25 13 16	41923	11·8		+ 6	− 3	8·8	23·01
50	+25 13 56	42419	10·1		+ 6	+ 3	8·7	23·03
51	+26 0 0	29*	8·3	C 14395	+ 3	+ 5	8·6	20·02
52	+26 14 48	36009	8·2		+ 5	+ 3	8·5	20·67
53	+25 13 48	42306*	9·0	B 4934	− 5	− 4	8·4	23·03
54	+26 14 48	36152	10·6		− 5	+ 3	8·4	20·67
55	+25 12 28	40847	9·8		+ 5	− 4	8·3	23·03
56	+25 12 28	40818*	6·8	B 4532	− 5	+ 4	8·3	23·03
57	+26 2 56	7315*	8·3	C 1528	− 4	− 5	8·3	23·15
58	+26 0 0	165	11·4		− 3	− 5	8·2	20·02
59	+25 13 48	42352	10·1		+ 6	− 2	8·2	23·03
60	+26 14 48	36088	10·9		− 4	+ 4	8·2	20·67
61	+25 13 48	42343	10·4		+ 2	+ 6	8·2	23·03
62	+26 0 0	35	11·4		− 2	− 5	8·1	20·02
63	+25 13 16	41905	10·7		− 6	+ 1	7·9	23·03
64	+25 13 40	42291*	9·0	C 6582	− 6	− 1	7·9	23·01
65	+25 13 56	42445	10·3		− 1	− 6	7·9	23·03
66	+25 13 24	42030*	8·2	C 6481	+ 6	0	7·8	23·00
67	+26 0 0	94*	6·7	C 14434	+ 5	+ 2	7·8	20·02
68	+25 13 16	41913	10·2		− 3	+ 5	7·6	23·01
69	+25 13 24	42009	10·5		+ 3	− 5	7·6	23·00
70	+26 2 56	7318*	9·3	C 1534	− 5	+ 3	7·6	23·15
71	+25 13 16	41925*	10·0	C 6415	+ 4	− 4	7·4	23·01
72	+25 13 40	42281	10·3		− 4	+ 4	7·4	23·01
73	+25 13 24	42041	11·2		+ 4	+ 4	7·4	23·00
74	+26 14 48	36155*	7·2	C 7005	− 1	+ 5	7·4	20·67
75	+25 12 28	40816*	8·5	B 4562	− 4	+ 4	7·4	23·01
76	+26 14 48	36036	10·3		− 3	+ 4	7·3	20·67
77	+26 14 48	36038	10·6		− 4	+ 3	7·3	20·67

TABLE II.—*continued.*

Ref. No.	Plate Centre.	Oxford. No.	Oxford. φ Mag.	A.G.C. No.	Δx. (Unit=0″·3.)	Δy. (Unit=0″·3.)	Cent. P.M. in Arc.	Interval in Years.
	° h m						″	
78	+26 14 48	36159	10·7		+ 4	− 3	7·3	20·67
79	+25 13 40	42258	11·0		− 5	− 2	7·0	23·01
80	+25 13 48	42360*	9·9	C 6660	+ 2	− 5	7·0	23·03
81	+25 13 48	42314*	9·8	B 4908	+ 5	− 2	7·0	23·03
82	+25 13 56	42449	10·3		− 5	− 2	7·0	23·03
83	+25 13 56	42477*	9·7	C 6682	+ 2	− 5	7·0	23·03
84	+25 13 56	42406	9·7		− 2	− 5	7·0	23·03
85	+25 13 24	42006	10·8		− 5	− 2	7·0	23·00
86	+26 2 56	7116	10·3		− 2	− 5	7·0	23·15
87	+26 2 56	7172	12·1		+ 5	− 2	7·0	23·15
88	+25 12 28	40853	11·3		+ 5	− 1	6·7	23·01
89	+25 13 40	42245*	10·6	C 6608	+ 1	+ 5	6·7	23·01
90	+25 13 24	42040	10·5		+ 5	+ 1	6·7	23·00
91	+25 13 56	42448*	8·1	C 6693	− 1	− 5	6·6	23·03
92	+25 13 48	42348	9·8		+ 5	+ 1	6·6	23·03
93	+25 13 48	42318	10·1		+ 1	+ 5	6·6	23·03
94	+25 12 28	40948	11·1		− 5	0	6·5	23·01
95	+25 13 40	42233*	9·3	C 6600	− 5	0	6·5	23·01
96	+26 2 56	7167	11·9		+ 3	− 4	6·5	23·15
97	+25 13 16	41926	12·0		+ 4	+ 3	6·5	23·01
98	+25 13 16	41935	11·8		+ 3	+ 4	6·5	23·01
99	+25 13 24	42039	11·2		+ 3	+ 4	6·5	23·00
100	+25 13 56	42434*	9·5	C 6704	− 4	+ 3	6·5	23·03
101	+25 13 48	42307	9·8		+ 5	0	6·5	23·03
102	+25 13 48	42315	9·8		+ 3	+ 4	6·5	23·03
103	+25 13 48	42317	10·1		0	− 5	6·5	23·03
104	+25 13 48	42330	9·8		+ 3	+ 4	6·5	23·03
105	+25 13 48	42346	10·1		− 3	+ 4	6·5	23·03
106	+25 13 48	42370*	9·5	C 6635	+ 4	+ 3	6·5	23·03
107	+26 14 48	36153*	9·6	C 6982	− 2	+ 4	6·3	20·67

The first star in Table II. (+26°, 132 Oxford) is the well-known triple star 85 Pegasi; see notes in the *Oxford Astrographic Catalogue*, viii. p. 125, and in Boss's *Catalogue*, p. 278. Boss's P.M.'s. are +ˢ·0622 and −0″·986, the corresponding Oxford results give +ˢ·0581 and −0″·975.

The collective result is one star in 44 with a C.P.M. greater than 15″, and 1 star in 12 with a C.P.M. between 15″ and 6″·3; approximately 2 and 8 per cent. of the number of stars examined.

University Observatory, Oxford:
1922 November 4.

The Problem of Disturbed Motion about the Equilateral Triangle Equilibrium Point and its Bearing on the Goldsbrough Theory of Saturn's Ring. By W. M. H. Greaves.

Suppose a planet J of mass m describes a circular orbit around the sun S of unit mass. Let B be the vertex of one of the two equilateral triangles described on SJ as base in the plane of motion of S and J. It is well known that B is a possible position of relative equilibrium for an asteroid which is subject to the gravitational influence of S and J.

The object of this paper is to discuss the behaviour of an asteroid A of infinitesimal mass, moving in the immediate neighbourhood of B, when the disturbing influence of an exterior planet P is taken into consideration. We shall confine ourselves to the case in which all the bodies move in the same plane. It will be shown that in general this disturbing influence of P is not of such a nature as to displace A permanently from the vicinity of the equilibrium point B, but that for certain ranges of values of the radius of the orbit of P, it may happen that A tends to move away from B. But, as will be pointed out, this result must be accepted with reserve, for our analysis will be based on a first approximation only, and it may happen that when the terms arising from higher approximations are taken into consideration, it will be found that A does not tend to depart irrevocably from the immediate vicinity of B.

We shall find that the case of the Sun, Jupiter, Saturn, and a Trojan asteroid moving in the neighbourhood of the equilibrium point corresponding to the Sun and Jupiter falls under the general case—that is to say, our work will indicate that the asteroid will not tend to depart irrevocably from the neighbourhood of the equilibrium point owing to the disturbing influence of Saturn.

We shall choose our units so that the numerical value of the constant of gravitation is unity.

Let $SJ = a$, $SP = a'$, and let n and n' be the angular velocities of J and P in their orbits round S, which are assumed circular. Let m' be the mass of P, it will be assumed that m' is small compared with m.

In assuming that the orbits of J and P are circular, we are to some extent neglecting their mutual perturbations. This is justifiable, for we are concerned with the motion of A, and those perturbations of A which depend on the mutual perturbations of J and P are of the nature of second order effects and may be neglected in a first approximation.

If P were absent, n would be given by the well-known relation,

$$n^2a^3 = 1 + m.$$

And if J were absent, n' would be given by

$$n'^2a'^3 = 1 + m'.$$

However, we shall not neglect the principal "secular" perturbation of P and J, which manifests itself in a modification of n and n'.

Taking this perturbation into account we have

$$\left.\begin{aligned} n &= a^{-\frac{3}{2}}\sqrt{1+m}(1+n_1) \\ n' &= a'^{-\frac{3}{2}}\sqrt{1+m'}(1+n_1')\end{aligned}\right\} \quad . \quad . \quad . \quad (1)$$

where *

$$\left.\begin{aligned} n_1 &= -m'a^2\frac{db^{(0)}}{da} \\ n_1' &= m\frac{\partial}{\partial a}(ab^{(0)})\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (2)$$

$b^{(0)}$ and α having the significance given below.

Let (r, θ) be the co-ordinates of A at time t.

The equations of motion of A are

$$\left.\begin{aligned} \ddot{r} - r\dot{\theta}^2 &= \frac{\partial V}{\partial r} \\ \frac{d}{dt}(r^2\dot{\theta}) &= \frac{\partial V}{\partial \theta}\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where

$$V = \frac{1}{r} + m\left\{\frac{1}{R} - \frac{r\cos \mathrm{ASJ}}{a^2}\right\} + m'\left\{\frac{1}{R'} - \frac{r\cos \mathrm{ASP}}{a'^2}\right\}$$

and

$$\mathrm{AJ} = \mathrm{R}, \qquad \mathrm{AP} = \mathrm{R}'.$$

Write

$$r = a(1+\rho)$$
$$\theta = nt + \sigma.$$

Then

$$\mathrm{ASJ} = \frac{\pi}{3} + \sigma$$

$$\mathrm{ASP} = (n-n')t + \sigma$$

$$\mathrm{R}^2 = r^2 + a^2 - 2ar\cos\left(\frac{\pi}{3} + \sigma\right)$$

$$\mathrm{R}'^2 = r^2 + a'^2 - 2a'r\cos[(n-n')t + \sigma]$$

[We have assumed that B precedes J in its motion. The corresponding results for the other equilibrium point B′ may be obtained by writing $\mathrm{ASJ} = \frac{\pi}{3} - \sigma$, ASP being $(n-n')t + \sigma$ as before.]

* See Tisserand, *Mécanique Céleste*, t. i. ch. xx. p. 347. The quantity $n_1 a'^{-\frac{3}{2}}$ is the predominant term in Tisserand's σ. n_1' may be found in a similar manner.

Write $$a = \frac{a}{a'}$$ and let

$$\{1 - 2a\cos\phi + a^2\}^{-\frac{1}{2}} = \tfrac{1}{2}b^{(0)} + b^{(1)}\cos\phi + b^{(2)}\cos 2\phi + \ldots + b^{(r)}\cos r\phi + \quad (4)$$

Neglecting the squares and products of ρ and σ, the equations (3) become

$$\left.\begin{aligned} a\ddot{\rho} - 2an\dot{\sigma} - an^2\rho - an^2 &= \left(\frac{\partial V}{\partial r}\right)_0 + a\rho\left(\frac{\partial^2 V}{\partial r^2}\right)_0 + \sigma\left(\frac{\partial^2 V}{\partial r\partial\theta}\right)_0 \\ a^2\ddot{\sigma} + 2a^2n\dot{\rho} &= \left(\frac{\partial V}{\partial\theta}\right)_0 + a\rho\left(\frac{\partial^2 V}{\partial r\partial\theta}\right)_0 + \sigma\left(\frac{\partial^2 V}{\partial\theta^2}\right)_0 \end{aligned}\right\} \quad (5)$$

The suffix o indicating that after the differentiations have been performed we put

$$r = a$$
$$\theta = nt.$$

Write $$\phi = (n - n')t \quad (6)$$

$$\kappa = \frac{1}{(n - n')a^{\frac{3}{2}}} \quad (7)$$

$$\kappa_1 = \frac{n}{n - n'} \quad (8)$$

On changing the independent variable from t to ϕ, (5) become

$$\left.\begin{aligned} \rho'' - 2\kappa_1\sigma' - \tfrac{2}{3}\kappa_1^2\rho - \kappa_1^2 &= \kappa_2 a^2\left(\frac{\partial V}{\partial r}\right)_0 + \kappa^2 a^3\left(\frac{\partial^2 V}{\partial r^2}\right)_0\rho + \kappa^2 a^2\left(\frac{\partial^2 V}{\partial r\partial\theta}\right)_0\sigma \\ \sigma'' + 2\kappa_1\rho' &= \kappa^2 a\left(\frac{\partial V}{\partial\theta}\right)_0 + \kappa^2 a^2\left(\frac{\partial^2 V}{\partial r\partial\theta}\right)_0\rho + \kappa^2 a\left(\frac{\partial^2 V}{\partial\theta^2}\right)_0\sigma \end{aligned}\right\} \quad (9)$$

differentiations with respect to ϕ being indicated by dashes.

Evaluating the quantities

$$\left(\frac{\partial V}{\partial r}\right)_0,\quad \left(\frac{\partial V}{\partial\theta}\right)_0,\quad \left(\frac{\partial^2 V}{\partial r^2}\right)_0,\quad \left(\frac{\partial^2 V}{\partial r\partial\theta}\right)_0,\quad \left(\frac{\partial^2 V}{\partial\theta^2}\right)_0$$

and substituting in (9) we obtain *

$$\left.\begin{aligned} &\rho'' - 2\kappa_1\sigma' + (\Phi_{10} + \Phi_{11}\cos\phi + \Phi_{12}\cos 2\phi + \ldots + \Phi_{1r}\cos r\phi + \ldots)\rho \\ &\quad + (\Phi_{20} + \Phi_{21}\sin\phi + \Phi_{22}\sin 2\phi + \ldots + \Phi_{2r}\sin r\phi + \ldots)\sigma \\ &\quad = \Phi_{30} + \Phi_{31}\cos\phi + \Phi_{32}\cos 2\phi + \ldots + \Phi_{3r}\cos r\phi + \ldots \\ &\sigma'' + 2\kappa_1\rho' + (\Phi_{40} + \Phi_{41}\sin\phi + \Phi_{42}\sin 2\phi + \ldots + \Phi_{4r}\sin r\phi + \ldots)\rho \\ &\quad + (\Phi_{50} + \Phi_{51}\cos\phi + \Phi_{52}\cos 2\phi + \ldots + \Phi_{2r}\cos r\phi + \ldots)\sigma \\ &\quad = \Phi_{60} + \Phi_{61}\sin\phi + \Phi_{62}\sin 2\phi + \ldots + \Phi_{6r}\sin r\phi + \ldots \end{aligned}\right\} \quad (10)$$

* The algebra omitted here is long but straightforward.

where

$$\left.\begin{aligned}
\Phi_{10} &= -\kappa_1^2 - \kappa^2(2-\tfrac{1}{4}m) - \tfrac{1}{2}\kappa^2 a^3 m' \frac{d^2 b^{(0)}}{da^2} \\
\Phi_{1r} &= -\kappa^2 a^3 m' \frac{d^2 b^{(r)}}{da^2} \quad (r \neq 0) \\
\Phi_{20} &= \Phi_{40} = -\frac{3\sqrt{3}}{4}\kappa^2 m \\
\Phi_{21} &= \Phi_{41} = -\kappa^2 a^2 m' \left(1 - \frac{db^{(1)}}{da}\right) \\
\Phi_{2r} &= \Phi_{4r} = \kappa^2 a^2 m' r \frac{db^{(r)}}{da} \quad (r \neq 0,\ 1) \\
\Phi_{30} &= \kappa_1^2 - \kappa^2(1+m) + \tfrac{1}{2}\kappa^2 a^2 m' \frac{db^{(0)}}{da} \\
\Phi_{31} &= \kappa^2 a^2 m' \left(\frac{db^{(1)}}{da} - 1\right) \\
\Phi_{3r} &= \kappa^2 a^2 m' \frac{db^{(r)}}{da} \quad (r \neq 0,\ 1) \\
\Phi_{50} &= -\tfrac{9}{4}\kappa^2 m \\
\Phi_{51} &= \kappa^2 a m' b^{(1)} - \kappa^2 a^2 m' \\
\Phi_{5r} &= \kappa^2 a m' r^2 b^{(r)} \quad (r \neq 0,\ 1) \\
\Phi_{60} &= 0 \\
\Phi_{61} &= \kappa^2 a m' (a - b^{(1)}) \\
\Phi_{6r} &= -\kappa^2 a m' r b^{(r)} \quad (r \neq 0,\ 1)
\end{aligned}\right\} \quad . \quad . \quad (11)$$

We shall confine ourselves to the complementary function of equations (10)—that is to say, we consider the equations,

$$\left.\begin{aligned}
&\rho'' - 2\kappa_1\sigma' + (\Phi_{10} + \Phi_{11}\cos\phi + \Phi_{12}\cos 2\phi + \ \ldots\ + \Phi_{1r}\cos r\phi + \ \ldots)\rho \\
&\qquad + (\Phi_{20} + \Phi_{21}\sin\phi + \Phi_{22}\sin 2\phi + \ \ldots\ + \Phi_{2r}\sin r\phi + \ \ldots)\sigma \\
&\qquad = 0 \\
&\sigma'' + 2\kappa_1\rho' + (\Phi_{40} + \Phi_{41}\sin\phi + \Phi_{42}\sin 2\phi + \ \ldots\ + \Phi_{4r}\sin r\phi + \ \ldots)\rho \\
&\qquad + (\Phi_{50} + \Phi_{51}\cos\phi + \Phi_{52}\cos 2\phi + \ \ldots\ + \Phi_{5r}\cos r\phi + \ \ldots)\sigma \\
&\qquad = 0
\end{aligned}\right\} \quad (12)$$

The "Particular Integral" of equations (10) can be found by the method used by Goldsbrough in his paper on "The Influence of Satellites upon the Form of Saturn's Ring." * It consists of periodic terms only.

We proceed to the discussion of equations (12).

Write

$$\left.\begin{aligned}
\rho &= x\cos\lambda - y\sin\lambda \\
\sigma &= x\sin\lambda + y\cos\lambda
\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (13)$$

where λ is given by $\tan 2\lambda = -\dfrac{2\Phi_{20}}{\Phi_{50} - \Phi_{10}}$ . . . . . (14)

* *Phil. Trans.*, A, 222, 116 and 117.

The equations (12) become

$$\left.\begin{aligned} x'' - 2\kappa_1 y' + \Theta_{10} x + m'(X_{11}\rho + X_{12}\sigma) &= 0 \\ y'' + 2\kappa_1 x' + \Theta_{50} y + m'(X_{21}\rho + X_{22}\sigma) &= 0 \end{aligned}\right\} \quad . \quad . \quad (15)$$

where

$$\left.\begin{aligned} \Theta_{10} &= \Phi_{10}\cos^2\lambda + 2\Phi_{20}\cos\lambda\sin\lambda + \Phi_{50}\sin^2\lambda \\ \Theta_{50} &= \Phi_{50}\cos^2\lambda - 2\Phi_{20}\cos\lambda\sin\lambda + \Phi_{10}\sin^2\lambda \end{aligned}\right\} \quad . \quad . \quad (16)$$

$$\left.\begin{aligned} X_{11} &= \theta_{11}\cos\phi + \psi_{11}\sin\phi + \theta_{12}\cos 2\phi + \psi_{12}\sin 2\phi + \ldots \\ &\qquad + \theta_{1r}\cos r\phi + \psi_{1r}\sin r\phi + \ldots \\ X_{12} &= \theta_{21}\sin\phi + \psi_{21}\cos\phi + \theta_{22}\sin 2\phi + \psi_{22}\cos 2\phi + \ldots \\ &\qquad + \theta_{2r}\sin r\phi + \psi_{2r}\sin r\phi + \ldots \\ X_{21} &= \theta_{41}\sin\phi + \psi_{41}\cos\phi + \theta_{42}\sin 2\phi + \psi_{42}\cos 2\phi + \ldots \\ &\qquad + \theta_{4r}\sin r\phi + \psi_{4r}\sin r\phi + \ldots \\ X_{22} &= \theta_{51}\cos\phi + \psi_{51}\sin\phi + \theta_{52}\cos 2\phi + \psi_{52}\sin 2\phi + \ldots \\ &\qquad + \theta_{5r}\cos r\phi + \psi_{5r}\sin r\phi + \ldots \end{aligned}\right\} \quad (17)$$

and

$$\left.\begin{aligned} m'\theta_{1r} &= \Phi_{1r}\cos^2\lambda + \Phi_{5r}\sin^2\lambda \\ m\,\psi_{1r} &= 2\Phi_{2r}\cos\lambda\sin\lambda \\ m'\theta_{2r} &= m'\theta_{4r} = \Phi_{2r}\cos^2\lambda \\ m'\psi_{2r} &= m'\psi_{4r} = (\Phi_{5r} - \Phi_{1r})\cos\lambda\sin\lambda \\ m'\theta_{5r} &= \Phi_{5r}\cos^2\lambda + \Phi_{1r}\sin^2\lambda \\ m'\psi_{5r} &= -2\Phi_{2r}\cos\lambda\sin\lambda \end{aligned}\right\} \quad . \quad . \quad (18)$$

The equations (15) are of the same type as a system of equations used by Goldsbrough in his paper upon the divisions in Saturn's Ring,* the only difference being that in Goldsbrough's equations the Ψ's are all zero. Goldsbrough's equations have been discussed by the writer in a paper which has appeared recently in the Cambridge Philosophical Society's *Proceedings*,† and by the same methods the following results applicable to the present case in which the Ψ's do not necessarily vanish can be established. ‡

x and y are expressible as sums of terms of the form $e^{c\phi}f(\phi)$, where the c's are constants and the $f(\phi)$'s are periodic functions of ϕ with period 2π.

The c's and the $f(\phi)$'s are expressible in the form of power series in m', convergent for sufficiently small values of m'.

* G. R. Goldsbrough, "The Influence of Satellites upon the Form of Saturn's Ring," *Phil. Trans.*, A, **222**, 109, equations (6), 1921.

† *Proc. Camb. Phil. Soc.*, **21**, 3, 281 to 289.

‡ The introduction of the Ψ's makes practically no difference. See Appendix.

Suppose we consider the solutions of (15) for different values of the parameter and which enters into the coefficients.

Then on the assumption that the roots of the quartic equation in v,

$$(\Theta_{10}-\nu^2)(\Theta_{50}-\nu^2)-4\kappa_1{}^2\nu^2=0 \quad . \quad . \quad . \quad (19)$$

are all real, it is found that (terms involving m'^2 and higher powers of m' being neglected)

(1) All the c's are purely imaginary quantities if all the quantities $\dfrac{m'}{\nu_r-\nu_s-m_{rs}}$ are small, where ν_r and ν_s are any two roots of (19), of which ν_r is the greater, and m_{rs} is the integer nearest to $\nu_r-\nu_s$. In general this condition will be satisfied and all the c's will be purely imaginary.

(2) But if for any value α_0 of α two roots of (19) differ by an integer, there will be a range of values of α, including α_0, for which some c's have real parts, the magnitudes of these real parts being of order m'.

Now if α be such that some of the c's are not purely imaginary, we are naturally led to suspect that the body A would depart from the neighbourhood of B on account of the presence of exponential factors $e^{c'\phi}$ in the expressions for x and y, where the c' are real quantities (some of which are positive). It must be remembered, however, that the equations (10) are themselves a first approximation, as they have been obtained by neglecting squares and products of ρ and σ, and it may happen that the terms involving $e^{c'\phi}$ may be the first terms in the expansion of a periodic function of ϕ in the rigorous solution of the problem.

Consider equation (19). It may be written

$$\nu^4-(\Theta_{10}+\Theta_{50}+4\kappa_1{}^2)\nu^2+\Theta_{10}\Theta_{50}=0.$$

Now from (16)

$$\Theta_{10}+\Theta_{50}=\Phi_{10}+\Phi_{50},$$

also

$$\Theta_{50}-\Theta_{10}=(\Phi_{50}-\Phi_{10})\cos 2\lambda-2\Phi_{20}\sin 2\lambda$$

$$=\frac{\Phi_{50}-\Phi_{10}}{\cos 2\lambda} \text{ from (14).}$$

$$\therefore\ 4\Theta_{10}\Theta_{50}=(\Phi_{10}+\Phi_{50})^2-(\Phi_{50}-\Phi_{10})^2\sec^2 2\lambda$$

$$=4\Phi_{10}\Phi_{50}-(\Phi_{50}-\Phi_{10})^2\tan^2 2\lambda$$

$$=4(\Phi_{10}\Phi_{50}-\Phi_{20}{}^2),$$

so that $\Theta_{10}\Theta_{50}=\Phi_{10}\Phi_{50}-\Phi_{20}{}^2$,

and (19) becomes

$$\nu^4-(\Phi_{10}+\Phi_{50}+4\kappa_1{}^2)\nu^2+\Phi_{10}\Phi_{50}-\Phi_{20}{}^2=0,$$

i.e.

$$(\Phi_{10}-\nu^2)(\Phi_{50}-\nu^2)=4\kappa_1{}^2\nu^2+\Phi_{20}{}^2 \quad . \quad . \quad . \quad (19)$$

In order to locate the regions in which some of the c's have real parts, let us put

$$m' = 0.$$

Then from (1)

$$n^3 a^3 = 1 + m,$$

and from (7) and (8)

$$\kappa_1^2 = \kappa^2(1 + m).$$

We then get from (11)

$$\Phi_{10} = -3\kappa^2(1 + \tfrac{1}{4}m)$$

$$\Phi_{20} = \Phi_{40} = -\frac{3\sqrt{3}}{4}\kappa^2 m$$

$$\Phi_{50} = -\frac{9}{4}\kappa^2 m,$$

and (19′) becomes

$$\nu^4 - \kappa^2\nu^2(1 + m) + \frac{27}{4}m\kappa^4 = 0 \quad . \quad . \quad . \quad (20)$$

The roots of (20) are $\pm\nu_1$, $\pm\nu_2$ where

$$\left.\begin{aligned} \nu_1^2 &= \kappa^2\frac{1 + m + \sqrt{(1+m)^2 - 27m}}{2} \\ \nu_2^2 &= \kappa^2\frac{1 + m - \sqrt{(1+m)^2 - 27m}}{2} \end{aligned}\right\} \quad . \quad . \quad . \quad (21)$$

ν_1 and ν_2 are real if

$$(1 + m)^2 > 27m \quad . \quad . \quad . \quad . \quad (22)$$

For any value of m satisfying (22), the roots of (20) will be real for sufficiently small values of m'.

For Jupiter $m = \frac{1}{1048}$ so that (22) is satisfied.

There are four cases in which two roots of (20) may differ by an integer:

(1) If $2\nu_1$ is an integer.
In this case the roots $+\nu_1$ and $-\nu_1$ differ by an integer.

(2) If $2\nu_2$ is an integer.
In this case the roots $+\nu_2$ and $-\nu_2$ differ by an integer.

(3) If $\nu_1 + \nu_2$ is an integer.
In this case the roots ν_1 and $-\nu_2$ differ by an integer, and the roots ν_2 and $-\nu_1$ differ by an integer.

(4) If $\nu_1 - \nu_2$ is an integer.
In this case the roots ν_1 and ν_2 differ by an integer, and the roots $-\nu_2$ and $-\nu_1$ differ by an integer.

If for any value a_0 of a, any one of the cases (1) to (4) occurs, then there will be a range of values of a, differing from a_0 by quantities of

order m', for which factors of the form $e^{c\phi}$ where c is real appear in the expressions for x and y. For other values of α, the expressions for x and y consist of periodic terms only.

For the actual case of Jupiter and Saturn m is small, and we have roughly,

$$n^2a^3 = 1$$

$$n'^2a'^3 = 1,$$

so that

$$\kappa = \kappa_1 = \frac{n}{n-n'}.$$

Furthermore, for Jupiter and Saturn $5n' = 2n$ roughly.
This gives

$$\kappa = \frac{5}{3}.$$

We see at once that this value of κ does not lie in the neighbourhoods of the values κ_0 (corresponding to the values α_0) for which exponential terms with real exponents c appear in the solution.

So that our work, as far as it goes, indicates that the disturbing influence of Saturn will not cause an asteroid situated near the equilateral triangle equilibrium point B corresponding to the Sun and Jupiter to depart from that neighbourhood.

The above theory is very similar to the Goldsbrough theory of the divisions in Saturn's Rings.* We have found that there are certain ranges of values of κ (or α) for which exponential factors $e^{c\phi}$, where c is real and positive, appear in the solution of the differential equations of motion. This also happens in the Goldsbrough theory. Now Goldsbrough's equations of motion, like the equations of motion in this paper, are themselves a first approximation, and it has been questioned † whether we should find any evidence of real instability on proceeding to the solution of the complete equations.

In the case of Goldsbrough's theory the problem is probably too complex to admit of a successful attempt at settling this question, but in the simpler problem we have been discussing it ought to be possible to go further into the matter, and examine the effect of the terms which were neglected in forming the equations. The mathematical analogy between the problem treated above and that discussed by Goldsbrough is so close, that the answer to the former would lead us to suspect the answer to the latter. Such an investigation would shed light on the Goldsbrough theory.

The best method of treatment would probably be to attack the problem of "Disturbed motion about the equilateral triangle equilibrium point" in another way, for example, by a Delaunay process. The writer hopes to return to this matter in a subsequent paper.

* *Phil. Trans.*, A, 222, 101–130, 1921.
† *Monthly Notices, R.A.S.* 82, 6, 356 to 359.

Appendix.

The effect of the ψ's on Equations (15).—When the ψ's vanish we get Goldsbrough's equations which have been discussed in the paper referred to above. The only difference that the presence of the ψ's makes is—that the expressions for h and h' in equations (15) of the paper in the Cambridge Philosophical *Proceedings* are replaced by

$$h = h_1 + h_2 i$$
$$h' = h_1' - h_2' i,$$

where h_1 and h_1' are the same as h and h' of the Cambridge Philosophical Society paper, being given by (15) of that paper, and

$$h_2 = \tfrac{1}{2}(\Theta_{50} - \nu_1^2)\psi_{1m} - \frac{(\Theta_{50} - \nu_1^2)(a_{10} - \nu_2^2)}{4\kappa\nu_2}\psi_{2m}$$
$$+ \kappa\nu_1\psi_{4m} + \frac{\nu_1}{2\nu_2}(a_{10} - \nu_2^2)\psi_{5m},$$
$$h_2' = \tfrac{1}{2}(\Theta_{50} - \nu_2^2)\psi_{1m} + \frac{(\Theta_{50} - \nu_2^2)(a_{10} - \nu_1^2)}{4\kappa\nu_1}\psi_{2m}$$
$$- \kappa\nu_2\psi_{4m} + \tfrac{1}{2}\frac{\nu_2}{\nu_1}(a_{10} - \nu_1^2)\psi_{5m},$$

the notation being that of the Cambridge Philosophical Society paper. We then get in place of (18) of that paper

$$\Delta = \Delta_1 + i\Delta_2,$$

where

$$\Delta_1 = 4ff'(h_1h_1' + h_2h_2') + (fg' - f'g)^2$$
$$\Delta_2 = 4ff'(h_1'h_2 - h_2'h_1).$$

If Δ_2 is not zero, c_1 will have a real part.

Note on the Proper Motions of Stars of the Clusters h and χ Persei.

By Rev. H. E. Macklin, S.J., B.Sc.

(*Communicated by Professor H. H. Turner.*)

In a previous paper, March 1921, I gave a résumé of these Proper Motions, obtained by a comparison with the plates measured by Mlles. Bronsky and Stebnitzky, and calculated over an interval of 25 years. These Proper Motions were relative, not absolute, so that it was important to compare them with those obtained by other observers.

The chief comparison was with the Proper Motions found by Professor Van Maanen, given in their revised form in *Contributions from the Mount Wilson Observatory*, Nos. 204 and 205. This comparison embraced

1030 stars contained in the two lists. The resultant residuals in units of 0·001 of a second of arc per annum are given below, arranged in groups covering 0·005 seconds of arc.

	In x or R.A. Direction.	In y or Dec. Direction.
Magnitude of Residual.	Number.	Number.
−57 to −53	4	1
−52 ,, −48	1	2
−47 ,, −43	2	1
−42 ,, −38	4	3
−37 ,, −33	23	3
−32 ,, −28	23	17
−27 ,, −23	52	20
−22 ,, −18	103	52
−17 ,, −13	106	67
−12 ,, − 8	126	136
− 7 ,, − 3	142	134
− 2 ,, + 2	143	149
+ 3 ,, + 7	106	142
+ 8 ,, +12	81	109
+13 ,, +17	46	83
+18 ,, +22	35	57
+23 ,, +27	13	29
+28 ,, +32	8	14
+33 ,, +37	4	4
+38 ,, +42	2	5
+43 ,, +47	1	1
+48 ,, +52	...	2
+53 ,, +57	1	...

These gave as the mean and probable error of a Proper Motion (the mean error being of the differences H.M. Proper Motion − Van Maanen Proper Motion):

$$\text{In } x - 0''\cdot005 \pm 0''\cdot00996 \text{ per annum.}$$
$$\text{,, } y + 0''\cdot0004 \pm 0''\cdot00962 \text{ ,,}$$

The smallness of the probable error is interesting in that the comparison was between relative and absolute Proper Motions, and between values derived from entirely different sets of measurements.

Another examination was made of the differences between the Proper Motions obtained by the comparison of my measures with those of Mlles. Bronsky and Stebnitzky, and the Proper Motions derived from my measures and those of the Rutherfurd plates. Here the agreement

was very close, the mean and probable errors (taken as before) of a Proper Motion being:

In x + 0″·0045 ± 0″·0073 per annum.
„ y − 0″·0010 ± 0″·0058 „

Finally, the Proper Motions derived from the comparison of my measures with the measures of Mlles. Bronsky and Stebnitzky were compared with those given in the paper of Professor Van Maanen mentioned above, obtained from meridian observations at the Dearborn Observatory. Here, as might be expected, the agreement is not so close, the mean and probable errors, expressed as before, being:

In x − 0″·0016 ± 0″·0142 per annum.
„ y + 0″·0005 ± 0″·0146 „

In the comparison with the Proper Motions found by Professor Van Maanen there were a number of larger discordancies. These are given below. The first column contains the Bronsky and Stebnitzky number, and the second the difference (excess of my Proper Motion over that found by Professor Van Maanen) expressed in units of 0·001 seconds of arc per annum in the x and y directions.

B.S. No.	M. − V. M.		B.S. No.	M. − V. M.	
	x	y		x	y
6	−26	+48	577	− 51	− 0
21	− 9	+67	810	− 56	−11
25	−15	+48	850	− 29	+44
32	− 7	−41	863	+ 41	+13
71	−42	+24	954	− 21	−48
91	−54	+ 5	1100	− 56	+ 1
343	− 1	−42	1252	+ 41	+ 3
359*a*	−30	−49	1313	−119	+ 8
372	−45	−11	1343	− 44	+16
453*a*	− 6	−53	1440	− 56	−12
488	+43	−49	1681	+ 54	+15
491	+16	−47			

Of these, the large difference for No. 1313 is clearly attributable to a misprint in the Bronsky and Stebnitzky catalogue, and should be added to the list of such given before. The remainder may be merely accidental errors, but are rather large.

These comparisons give an idea of the standard of accuracy to be expected in the photographic determination of Proper Motions, which is certainly higher, in general, than that attained in Proper Motions derived from meridian observations; also it seems to show that if enough stars be taken in the reduction of the two sets of star positional measures, the difference between absolute and relative Proper Motions is almost negligible.

Note on a Paper by M. Shajn. By E. M. Antoniadi.

On p. 544, vol. lxxxii., of the *M.N.*, the Secretaries have called attention to a paper by M. Shajn, published in Russian, in the *Bulletin de la Société astronomique de Russie* for 1912, in which he is alleged to have reached "much the same conclusions" as the present writer in 1916 "on the co-relation of the rate of melting of the Martian Polar Caps with the variation of sunspot activity."

Of course, the writer never saw this paper, or even any allusion to it. He does not know the Russian language; never heard of the author in question; and was ignorant, up to now, even of the existence of a Russian astronomical association. It is curious that in the annual review of works received by the Society prior to 1916 the writer could find no trace of the above *Bulletin*. It is also singular that the able and exhaustive yearly reviews on solar physics in *M.N.*, signed "A. F." and "F. J. M. S.," are silent, from 1912 to 1916 at least, on the subject of the Russian memoir. If, then, those having access to the astronomical publications of the whole world failed to take notice of M. Shajn's paper, the chances were against the writer's seeing it, considering the modesty of his library and the fact that in 1912 he was receiving only four astronomical serials of Western Europe.

The writer's inquiry into this question was bound to come as a sequel to his measures of the size of the Martian snow-caps on all drawings since 1856 and his exhaustive study of the polar whitishnesses or haze.* Having recognised that these whitishnesses are more frequent in the N. frigid zone than in the S. one, which confirms the fact that haze on Mars develops more readily over the yellow areas than over the grey ones; that the S. cap emerges larger than the N. one from the polar night, as forming in a longer winter; and that the detachment of Olympia from the N. snows, or of Novissima Thyle from the S. ones, is a phenomenon of yearly occurrence on the planet—he next found the diminution of the polar snows to be erratic. On considering the possible causes of such a deportment, the idea occurred to him that the melting of the Martian snow-caps could be subordinated to possible changes in the heat of the Sun. A careful treatment of the subject led to the confirmation of this suspicion, when the variations in the intensity of solar radiation, as evidenced by the diminution of the snows of the planet Mars, appeared as a small percentage of the whole.

The note by the Secretaries prompts the writer gladly to accord M. Shajn a priority of four years in the conception and probable demonstration of the relation in question. Yet the writer maintains intact the originality of his work and conclusions, and wishes to lay stress on the discrepancies due to changes in the atmosphere of Mars, leading to very marked divergences in the two curves, whereas those of M. Shajn are described as "nearly parallel." Thus the writer could find no agreement in 1862, 1873, and 1877 at least; and the micrometric

* These results have been embodied abridged in the *Mars Report, B.A.A.*, for 1911–12 (published in 1916), and shaped into a permanent ephemeris.

measures of the S. snow-cap by Dr. Barnard with the 36-in. Lick refractor in 1892 point to a slow melting at a time of almost maximum solar activity. Lastly, since M. Shajn's work stops in 1907, the present writer's demonstration that the melting of the Martian snow was slowest during the virtually unprecedented minimum of sunspots of 1911–1913 remains also absolutely original and cannot be wrested from him.

Paris:
1922 *November* 20.